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DISCUSSION PAPER

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# Technology Licensing and the Direction of Innovation

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# Technology Licensing and the Direction of Innovation*

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**Abstract**

Previous work shows that when firms choose the direction of their research before how much to invest, a generous patent regime can induce them to inefficiently converge on the same technological area. We ask whether technology licensing, via fixed fees or per-unit royalties, corrects this failure. The answer rests on cross-licensing: under both contracts, the prospect of combining complementary innovations developed along different trajectories can lead firms to differentiate. In a Cournot illustration, the inefficiency survives only when innovations are close substitutes and products are not too differentiated: then the cross-license adds little to what each firm already owns or, when one firm invests more, research along different trajectories becomes a contest to supply the technology both firms use. In the same illustration, lump-sum fees dominate per-unit royalties in terms of welfare. Technology markets and patentability requirements thus appear to be complementary instruments, and antitrust scrutiny should target the terms of cross-licensing agreements rather than the institution itself.

**Keywords:** direction of innovation; technology licensing; cross-licensing; fixed fees; royalties; patentability requirements.

**JEL codes:** L13, L24, O31, O34.

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# 1 Introduction

Innovation is a key driver of long-term economic growth and an essential tool in firm strategy. Traditionally, the literature on R&D has mostly focused on how competition affects the intensity of innovation, that is, firms' investments to increase the probability of discovery or the value of what is discovered (Arrow, 1962; d'Aspremont and Jacquemin, 1988; Suzumura, 1992; Aoki and Spiegel, 2009; Delbono and Lambertini, 2020). More recently, scholars have begun to investigate why firms select specific technological trajectories and what determines the direction of innovation (Bryan and Lemus, 2017; Hopenhayn and Squintani, 2021; Dijk et al., 2024; Teshima, 2026). Within this literature, Manenti and Sandrini (2025) (henceforth MS2025) show that the direction firms choose responds to the characteristics of the patent regime: in a duopoly with uncertain R&D outcomes, where firms choose a technological trajectory before investing, a generous patent regime, i.e., one under which even low-value innovations are patentable, can induce firms to concentrate on the same technological domain. Such convergence is socially undesirable: firms invest less than they would along differentiated trajectories, and the ensuing patent race wastefully duplicates R&D efforts. MS2025 show that stricter patentability requirements can eliminate this directional inefficiency.

That analysis — and, more broadly, the literature on the direction of innovation — abstracts from technology licensing. The omission is not innocuous. Markets for technology are pervasive and economically large (Arora and Gambardella, 2010; Rostoker, 1984), and licensing is known to reshape innovation incentives, both when the innovator is an outside laboratory (Kamien and Tauman, 1986; Kamien et al., 1992; Stamatopoulos and Tauman, 2008) and when, as here, the innovator competes in the product market against its potential licensee (Katz and Shapiro, 1985; Gallini and Winter, 1985; Wang, 1998; Sen and Tauman, 2007; Jeon and Lefouili, 2018). If a patent can be monetized not only through the product market but also by selling access to the rival, both the value of innovating and the relative value of a firm's technological choice change. This paper asks whether the market for technology corrects the directional failure identified by MS2025, and whether the answer depends on the form of the licensing contract.

We study two firms that sell differentiated products and invest in incremental, product-enhancing innovations, each of uncertain success. Firms first choose a technological trajectory, then decide how much to invest in it. Once an innovation is developed successfully, we assume that it is eligible for patent protection and focus on a generous patent regime (i.e., one in which the minimum value required for patentability is low enough that every successful innovation qualifies for protection). This environment is, indeed, where the market failure identified by MS2025 arises. As in MS2025, the direction of research matters because it determines whether both innovations can be patented. When firms succeed on overlapping trajectories, their innovations cover the same technological territory and cannot both be protected, so that a single patent is granted; on non-overlapping trajectories, both innovations are patented and the two patents coexist. We depart from MS2025 by letting patents be licensed: through one-way licensing when a single firm holds a patent, and through paid cross-licensing when both firms succeed on non-overlapping trajectories. We consider two contractual forms: lump-sum

fixed fees and per-unit royalties.

Modelling licensing adds a new dimension relative to MS2025. There, what matters about innovations is only whether they belong to the same technological domain, as this is what determines whether both can be patented. Once technologies can be traded, however, two innovations that are technologically non-overlapping (and hence both patentable) may be worth very different amounts to the rival firm, depending on how effectively and/or profitably a firm can "absorb" the licensed technology into its own production process or product. We capture this with a single measure of the complementarity between the two innovations. At one extreme, two innovations developed in distinct technological domains may be pure substitutes for the firms: think of two smartwatch producers extending battery life, one through a higher-density battery, the other through software power management. These are distinct technologies, hence both patentable, yet they improve the final good along the same product attribute, so that a firm gains nothing from adding the rival's technology to its own, and access to the rival's patent is valuable only if the rival's technology is better, in which case the firm can replace its own. At the other extreme, innovations may be complements: a low-power display and a low-latency connectivity module, or a better biometric sensor and a better algorithm to interpret its data. In this case, innovations are worth more when combined than taken alone.[1] Cross-licensing is precisely the institution through which this complementarity is priced and traded.

Our message is that licensing reduces the directional inefficiency identified in MS2025, under both contractual forms. Three sets of results support it. First, on innovation intensity, neither fixed-fee nor per-unit royalty licensing overturns the investment ranking of MS2025: firms keep investing more along non-overlapping trajectories than when developing overlapping technologies. Second, on direction of innovation: we show that the choice of R&D trajectory depends on the degree of complementarity between innovations. With fully complementary innovations, cross-licensing induces firms to differentiate their R&D projects rather than converge on the same technological trajectory, unless the cost of the additional R&D it triggers dissipates the gain from combining the two technologies. Moving away from full complementarity, firms keep differentiating as long as complementarity is high enough, provided, as above, that the gain from a more complementary cross-license is not offset by the firms' investment response. Third, on welfare: as in MS2025, differentiation remains the socially preferred direction once technologies can be licensed; what licensing changes is where firms fail to choose it. The directional inefficiency is confined to low degrees of complementarity, where firms converge. In our illustrative Cournot model, this happens only for innovations that are close substitutes and products that consumers do not regard as too differentiated.

The force behind these results is cross-licensing. One-way licensing takes place whatever the direction of research, whenever a single firm holds a patent; cross-licensing requires two patents on distinct technologies, and is therefore available only to firms that differentiate, which can

[1] The logic is not confined to high-tech industries. Two premium-yogurt producers may both pursue creaminess, one through a new bacterial culture and the other through mechanical filtration (non-overlapping technologies, substitute innovations), or one may develop a low-sugar formulation while the other improves shelf-life packaging (non-overlapping and complementary).

then combine their innovations.

Close innovation substitutability also brings a new force into play. A firm whose technology is substantially better than the rival's can license it to a rival that replaces its own: investing more than the rival then pays off twice, through a better product and through the sale of the whole technology. When complementarity is low, this incentive breaks down the symmetric investment equilibrium, and an asymmetric one emerges, in which one firm invests more and the other adopts its technology when both succeed. Firms then no longer compete to combine their technologies, but over whose technology both of them will use. In this scenario, cross-licensing no longer serves to combine the two technologies, and the directional inefficiency may reappear.

Our results complement those of MS2025: a well-functioning market for technology and stricter patentability requirements address the directional failure in different industries, the former where innovations along different trajectories are complementary, the latter where they are close substitutes. From an antitrust perspective, the institution of cross-licensing in the presence of running royalties has been under scrutiny for its potential anti-competitive effect (see Jeon and Lefouili, 2018). While we do not investigate here the incentives of the firms to set royalties anti-competitively, our results suggest that antitrust scrutiny should focus on the terms of cross-licensing agreements rather than on the institution itself, since cross-licensing can steer R&D ex-ante towards the efficient differentiation of projects. As long as royalties do not exceed the value of the technologies exchanged, this benefit does not come at the cost of weaker competition ex post. The form of the payment, however, determines whether consumers share in the gains from complementarity: lump-sum fees leave margins higher and output larger, while royalties leave them unchanged.

Our contribution to the literature is twofold. To the literature on the direction of innovation (Bryan and Lemus, 2017; Hopenhayn and Squintani, 2021; Dijk et al., 2024; Manenti and Sandrini, 2025; Teshima, 2026), we add the market for technology, and show that it reduces the directional inefficiency of a generous patent system. To the literature on patent licensing between product-market rivals, which has largely focused on the intensity of innovation and on the comparison of contractual forms with an exogenous research direction (Katz and Shapiro, 1985; Gallini and Winter, 1985; Kamien and Tauman, 1986; Sen and Tauman, 2007; Fan et al., 2018; Hsu et al., 2019), we add an endogenous direction.

A close reference is Fershtman and Kamien (1992), who study how firms develop complementary technologies that may later be cross-licensed. Our setting differs from theirs in three main aspects: first, they consider a game where an innovation can only be patented once the firms have developed (or secured through licensing) two fundamental technologies. Here, we investigate different degrees of complementarity between successful innovations, which can either be combined or replaced by the competing firms. Second, our goal is the direction of innovation. Thus, we analyze the incentives of the firms to invest in overlapping or non-overlapping technologies. Instead, Fershtman and Kamien (1992) requires both technologies to market the final innovation. Finally, in our setting, the cross-licensing channel, well documented in prac-

tice (Grindley and Teece, 1997) and studied mainly for its product-market effects (Jeon and Lefouili, 2018), shapes R&D choices ex ante, before any innovation exists. Unlike the literature on patent pools and blocking patents (Shapiro, 2001; Lerner and Tirole, 2004), each patent in our model can be exploited independently of the other, and cross-licensing is valuable because the two technologies can be combined or, when they are close substitutes, because the better one can replace the worse. Furthermore, in our paper, cross-licensing shapes endogenous ex-ante R&D trajectory rather than ex-post pricing of exogenous patent portfolios.

The remainder of the paper is organized as follows. Section 2 presents the model and the assumptions, common to both contractual forms. Sections 3 and 4 analyze fixed-fee and royalty licensing, respectively. Section 5 discusses near-substitute innovations and technology replacement, and closes with the policy implications. Section 6 confirms all results (and verifies all assumptions) within an illustrative Cournot model with non-drastic product innovation. Section 7 concludes. All proofs are in the Appendix.

# 2 The model

Following Manenti and Sandrini (2025), we consider two symmetric firms, $i$ and $j$, competing in the product market. Before competition takes place, firms engage in R&D. The innovation process consists of two decisions: firms first choose the technological territory (or project) in which to conduct their research, and then how much to invest in the chosen direction. There are two available trajectories; when both firms select the same one, we say that technologies are overlapping, while when they select different trajectories technologies are non-overlapping.[2] Investing $I(\beta_i)$, with $I(\cdot)$ increasing and convex, $I(0) = 0$ and $I'(0) = 0$, firm $i$ obtains, with exogenous probability $p \in (0, 1]$, an innovation of quality $\beta_i \geq 0$; with probability $1 - p$ the research project fails and the firm remains with the status-quo technology, whose quality is normalized to zero.

Patent protection follows MS2025. An innovation is patentable if its value exceeds a minimum requirement; we consider a generous patent regime, in which this requirement is low enough that every successful innovation can be patented. This is precisely the environment in which the directional inefficiency arises. When the two firms successfully innovate in different projects, both innovations are patented and the two patents coexist. When firms innovate within the same domain, the two innovations cannot both be protected: the patent office randomly assigns the patent to either firm with probability 1/2 (a patent lottery). If only one firm succeeds, it obtains the patent regardless of the direction chosen.[3,4]

We depart from MS2025 by allowing for technology licensing: a firm holding a patent can license its technology to the rival. Two licensing channels are open to firms:

[2]The analysis easily carries over to any general $m \geq 2$ number of projects, provided that they are all available to both firms.

[3]See Manenti and Sandrini (2025) for a detailed discussion of this assumption.

[4]With a slight abuse of notation, throughout the paper, we will sometimes refer to $\beta_i$ as the value of firm $i$'s innovation and the value of firm $i$'s patent, interchangeably.

(i) one-way licensing: when a single firm holds a patent (because it was the only successful innovator, or because it won the patent lottery), it can license its technology to the unsuccessful rival;

(ii) cross-licensing: when both firms succeed on non-overlapping trajectories, each holds a patent on a distinct technology and the two firms can license their innovations to each other.

Patents are not blocking: when both firms hold a patent, each can exploit its own technology whether or not a cross-license is signed, so that the outside option of each firm, when deciding whether to accept a license, is to keep operating the technology it owns. As a tie-breaking rule, we assume that a licensee indifferent between accepting and rejecting a license accepts it. The two sections that follow consider, in turn, two contractual forms: a lump-sum fixed fee (Section 3) and a per-unit royalty (Section 4).

All the assumptions introduced in this section are meant to be intuitive restrictions on reduced-form payoffs, consistent with the most commonly used models of oligopolistic competition; each of them is explicitly verified in the illustrative Cournot model presented in Section 6.

## 2.1 Payoffs

Payoffs in the product market, gross of the investment cost, depend on the value of firms' innovations. We write $\pi_i(\beta_i, \beta_j)$ for the payoff of firm $i$ when firms compete with technologies of value $\beta_i$ and $\beta_j$.[5] Importantly, $\pi_i$ increases in $\beta_i$ and decreases in $\beta_j$. When only one firm (say $i$) holds a patent and no licensing takes place, its payoff is $\pi_i(\beta_i, 0)$ and the rival earns $\pi_j(0, \beta_i)$; if both firms fail, payoffs are $\pi_i(0, 0)$. Our framework is symmetric: the two firms have the same payoff function, $\pi_i(x, y) = \pi_j(x, y)$ for any qualities $x$ and $y$, so that we drop the firm subscript whenever no confusion arises. The following two assumptions are taken directly from MS2025 and discipline payoffs in the absence of any licensing. They apply throughout the paper, to both contractual forms.

**Assumption 1** (On profits ranking)**.** *For a given common quality of firms' patents, $\beta$:*

$$\pi_i(\beta, 0) \;>\; \pi_i(\beta, \beta) \;>\; \pi_i(0, 0) \;\geq\; \pi_i(0, \beta) \;>\; 0, \qquad \forall \beta.$$

The intuition is straightforward. Being the sole patent holder is the most profitable position: the firm enjoys a quality advantage over an unimproved rival. Joint success is less profitable than exclusive success – the rival's innovation erodes part of the advantage – but still better than the status quo: a better technology pays even when the rival also improves. Finally, competing with no innovation against a patented rival is the worst outcome, though profits never become negative (the innovation is non-drastic).

[5] Throughout, the first argument of a payoff function refers to the quality of the firm whose payoff it is, and the second to that of its rival. So, $\pi_i(\beta_i, 0)$ and $\pi_j(0, \beta_i)$, for example, indicate the payoffs of firms $i$ and $j$, respectively, when $i$ is the only one holding a patent.

**Assumption 2** (On marginal benefits)**.** *For a given common quality of firms' patents,* $\beta$*:*

$$a)\ \left.\frac{\partial \pi_i(\beta_i, 0)}{\partial \beta_i}\right|_{\beta_i=\beta} > \left.\frac{\partial \pi_i(\beta_i, \beta)}{\partial \beta_i}\right|_{\beta_i=\beta}, \qquad b)\ \left.\frac{\partial \pi_i(\beta_i, 0)}{\partial \beta_i}\right|_{\beta_i=\beta} < 2\left.\frac{\partial \pi_i(\beta_i, \beta)}{\partial \beta_i}\right|_{\beta_i=\beta}.$$

Part $a$) says that a marginal improvement in quality is worth more when the advantage it confers is exclusive: when the rival has also innovated, part of the marginal gain is competed away. Part $b$) bounds the exclusivity advantage at the margin: the marginal return to an exclusive innovation cannot exceed twice the return when the rival also innovates.

The remaining two assumptions of this subsection extend the framework in MS2025 to the two licensing channels introduced above: the first is a natural extension of Assumptions 1–2 to one-way licensing; the second is specific to the possibility of cross-licensing.

**One-way licensing.** Licensing allows an unsuccessful firm to access and employ the successful technology $\beta$; when both firms operate technologies of common quality $\beta$ following such a transfer, firm $i$'s gross payoff (gross of any licensing payment) is denoted $\tilde{\pi}_i(\beta, \beta)$.

**Assumption 3.** *For a given common quality of firms' patents,* $\beta$*:*

$$a)\ \pi_i(\beta, \beta) \geq \tilde{\pi}_i(\beta, \beta) > \pi_i(0, 0); \qquad b)\ \left.\frac{\partial \pi_i(\beta_i, \beta)}{\partial \beta_i}\right|_{\beta_i=\beta} > \left.\frac{\partial \tilde{\pi}_i(\beta_i, \beta_i)}{\partial \beta_i}\right|_{\beta_i=\beta}.$$

Part $a$) states that licensing merely replicates an existing technology and does not expand the technological frontier: joint success on distinct trajectories weakly dominates the duplication of a single innovation through licensing.[6] Part $b$) is its counterpart at the margin: gross profits increase more with $\beta_i$ when a marginal improvement in quality remains exclusive rather than being shared with the rival through licensing. This is natural, as improving an innovation also improves the competitive position of the rival that is licensing in the innovation.

**Cross-licensing and the complementarity of innovations.** When both firms succeed on non-overlapping trajectories, each firm holds a patent on a distinct technology, and cross-licensing allows each firm to access and employ the rival's technology. A firm that licenses the rival's technology can add it to its own. How much this raises the quality of its product depends on how the two innovations, developed along different technological trajectories, relate to one another. When they are complements, the firm can incorporate the rival's innovation on top of its own; when they are substitutes, improving the same product attribute, adding one to the other yields little. We capture this with a single parameter $\theta \in [0, 1]$, the share of the rival's technology that a firm can absorb on top of its own: by adding the rival's technology, firm $i$ raises the quality of its product from $\beta_i$ to $\beta_i + \theta\beta_j$. Naturally, the firm always retains the option of giving up its own technology, even when enhanced with the rival's, and of operating the rival's technology alone, exactly as a firm that failed in its research does under a one-way

[6] In models of process innovation it is natural to have $\pi_i(\beta, \beta) = \tilde{\pi}_i(\beta, \beta)$, as firms face the same production costs under joint success and under licensing; the same equality obtains in the product-innovation illustration of Section 6, where licensing duplicates the demand-enhancing innovation.

license. We refer to this option as *replacement*: when it occurs, the quality of $i$'s product is $\beta_j$. This option is worthwhile only when the rival's technology is substantially better and the two innovations are close substitutes, so that adding the rival's technology to its own would yield little. Since the firm chooses the more valuable of the two options, under cross-licensing the effective quality of its product is

$$\hat{\beta}_i \;\equiv\; \max\{\beta_i + \theta\,\beta_j,\; \beta_j\}. \tag{1}$$

Replacement is optimal when $\beta_i < (1-\theta)\beta_j$. When $\theta = 1$ the two innovations are fully complementary: each firm can incorporate the rival's innovation in full on top of its own, and replacement never occurs. As $\theta$ decreases the two innovations become closer substitutes; when $\theta = 0$ they are perfect substitutes: each firm has, in effect, developed its own version of the same functionality, and access to the rival's technology is valuable only if the rival's version is better, in which case the firm replaces its own. Notice that at symmetric qualities replacement never occurs, and $\hat{\beta}_i = \hat{\beta}_j = (1+\theta)\beta$.[7]

We write $\tilde{\pi}_i^x(\hat{\beta}_i,\, \hat{\beta}_j)$ for firm $i$'s gross payoff when a cross-license agreement (indicated by the superscript $^x$) is in place, evaluated at the relevant product-market equilibrium. The following assumption disciplines how complementarity affects this payoff.

**Assumption 4.** *At any common quality $\beta > 0$, $\tilde{\pi}^x(\beta+\theta\beta,\, \beta+\theta\beta)$ is strictly increasing in $\theta$, with $\tilde{\pi}^x\big|_{\theta=0} = \pi(\beta,\beta)$.*

The intuition is immediate: the more complementary the two innovations, the more each firm gains from absorbing the rival's technology into its own – both firms operate better technologies, which (given that qualities enter symmetrically) raises their gross profits. At $\theta = 0$ and symmetric qualities cross-licensing is valueless and payoffs collapse to the no-licensing, both-success configuration. The assumption is verified in the illustrative model of Section 6.

Since cross-licensing is possible only when firms innovate along different trajectories, the degree of complementarity leaves the value of overlapping trajectories unaffected, while, by Assumption 4, it raises the value of non-overlapping ones. This asymmetry is the core of our analysis.

## 2.2 Consumer surplus and welfare

To study efficiency we now turn from payoffs to consumer surplus, imposing mild regularity conditions taken from MS2025 and extended, in turn, to one-way licensing and to cross-licensing. Let $CS(\beta_i, \beta_j)$ denote consumer surplus when firms compete with distinct technologies of qualities $\beta_i$ and $\beta_j$; let $\widetilde{CS}(\beta,\beta)$ denote consumer surplus when, following a one-way licensing agreement, both firms operate a common technology of quality $\beta$; and let $\widetilde{CS}^x(\beta,\beta)$ denote

[7] This formulation applies to both process and product innovations: in the first case, the licensee's marginal cost falls by $\hat{\beta}_i - \beta_i$; in the second, its demand intercept rises by $\hat{\beta}_i - \beta_i$. In the linear model of Section 6 the two cases are formally equivalent.

consumer surplus when both firms operate a common quality $\beta$ reached through cross-licensing – each, in every case, evaluated at the relevant product-market equilibrium.[8]

**Assumption 5.** *For a given common value of patentable innovation, $\beta$:*

*a)* $\dfrac{\partial CS(\beta_i,\beta)}{\partial \beta_i} = \dfrac{\partial CS(\beta,\beta_j)}{\partial \beta_j} > 0, \quad$ *and* $\quad \dfrac{\partial \widetilde{CS}(\beta,\beta)}{\partial \beta} > 0;$

*b)* $CS(\beta,\beta) \geq \widetilde{CS}(\beta,\beta);$

*c)* $\widetilde{CS}^x(\beta+\theta\beta,\beta+\theta\beta)$ *is non-decreasing in* $\theta$*, with* $\widetilde{CS}^x\big|_{\theta=0} = CS(\beta,\beta)$.

Part *a*) of Assumption 5, taken from MS2025, states that consumers benefit from higher innovation quality, whether reached directly or through one-way licensing. Part *b*), which extends MS2025 to licensing, compares two configurations in which both firms operate a technology of the same quality $\beta$: in the first, each firm owns a distinct patented innovation; in the second, one firm uses the rival's innovation under a one-way license. The assumption states that consumers are not worse off in the first configuration. This is natural for two reasons. First, a license transfers a technology but does not improve it, so that the licensee cannot offer consumers more than it would with an innovation of its own of the same quality; this is the consumer-side counterpart of Assumption 3*a*). Second, the licensing contract may itself distort product-market behavior: a per-unit royalty raises the licensee's marginal cost and reduces its output, to the detriment of consumers, whereas a lump-sum fee leaves output unaffected.[9] As in MS2025, differentiation is therefore the socially preferred direction; the question is whether firms' private incentives can be aligned with it. Part *c*) is the consumer-surplus counterpart of Assumption 4, and states that a more complementary cross-license (weakly) benefits consumers as well, collapsing to the no-licensing configuration at $\theta = 0$. Its weak form (i.e. non-decreasing) accommodates contractual arrangements in which the licensor appropriates, through the royalty, the entire value of the technology it transfers, leaving consumer surplus unchanged. All three parts are verified in the illustrative model of Section 6.

Throughout, we say that the private choice of direction is inefficient when firms choose overlapping trajectories while expected welfare, i.e. the sum of expected consumer surplus and expected industry profits net of R&D costs, would be higher under non-overlapping ones. In the general model, Assumption 5 allows us to rank consumer surplus across regimes; total welfare is computed explicitly in the illustrative model of Section 6.

# 3 Fixed-fee licensing

Consider first a lump-sum arrangement: licensing payments are fixed fees, paid upfront, that do not depend on the licensee's output.

[8] Under royalty licensing (Section 4), the relevant equilibrium quantities are distorted by the running royalty; $\widetilde{CS}$ and $\widetilde{CS}^x$ are computed at those quantities.

[9] In the illustrative model of Section 6, the assumption thus holds with equality under fixed fees and strictly under royalties. We rule out the case in which sharing the same technology makes competition so much tougher that the fall in prices more than compensates consumers.

### 3.1 One-way licensing

When a single firm, say firm $i$, holds a patent of quality $\beta_i$, it can license it to the rival against a fee $L(\beta_i)$, with $L(0) = 0$. Licensing arises if and only if the licensor is compensated for the loss of its competitive advantage and the licensee is better off paying the fee:

$$\underbrace{\pi_i(\beta_i, 0) - \tilde{\pi}_i(\beta_i, \beta_i)}_{\equiv \underline{L}(\beta_i)} \;\leq\; L(\beta_i) \;\leq\; \underbrace{\tilde{\pi}_j(\beta_i, \beta_i) - \pi_j(0, \beta_i)}_{\equiv \overline{L}(\beta_i)}. \tag{2}$$

The lower bound $\underline{L}$ is the licensor's minimum compensation and amounts to the profit it gives up by sharing its exclusive advantage. The upper bound $\overline{L}$ is the licensee's maximum willingness to pay, i.e. its net gain from accessing the technology. The band (2) is non-empty if and only if $2\tilde{\pi}(\beta, \beta) \geq \pi(\beta, 0) + \pi(0, \beta)$: licensing must raise gross industry profits. Otherwise, there is no contract that satisfies the incentive compatibility constraint of the licensor or the participation constraint of the licensee. If this is the case, licensing does not occur and we are back to MS2025. Our analysis explicitly focuses on market environments where licensing can occur, and we maintain this condition throughout, so that one-way licensing takes place whenever a single firm holds a patent.[10]

Throughout the paper, we assume that the patent holder makes a take-it-or-leave-it offer to the rival. The fee is therefore set at the licensee's maximum willingness to pay, $L(\beta) = \overline{L}(\beta)$, and the licensee is left at its outside option.

### 3.2 Cross-licensing

When both firms succeed on non-overlapping trajectories, each holds a patent on a distinct technology and can license it to the rival against a fixed fee, so that there are two licenses, one per direction of the technology exchange. Consider the license on firm $i$'s technology (to be used by firm $j$), taking the other direction of the exchange as given. Firm $i$'s minimum compensation is

$$\underline{L}_i^x = \pi_i(\hat{\beta}_i,\, \beta_j) - \pi_i(\hat{\beta}_i,\, \hat{\beta}_j), \tag{3}$$

which is the minimum compensation firm $i$ is willing to accept for licensing its own technology to firm $j$, given that firm $j$ already grants its own technology to firm $i$. Analogously, firm $j$'s maximum willingness to pay for firm $i$'s technology is its gain from accessing that technology, given that it already grants its own technology to firm $i$:

$$\overline{L}_i^x = \pi_j(\hat{\beta}_j,\, \hat{\beta}_i) - \pi_j(\beta_j,\, \hat{\beta}_i). \tag{4}$$

As in the one-way case, each patent holder makes a take-it-or-leave-it offer for its own technology, taking the other license as given, and sets the fee at the rival's maximum willingness

[10] In the illustrative model of Section 6, the condition holds for every innovation size when products are sufficiently differentiated and, otherwise, for innovations that are not too large (footnote 21). At the equilibrium investment levels, it holds for every degree of product substitutability and complementarity when $p \leq 1/2$.

to pay, $L_i^x = \overline{L}_i^x$ in (4).[11] The fee firm $j$ pays for firm $i$'s technology, $L^x(\beta_i, \beta_j)$, depends on both qualities: the first argument is the quality of the licensed technology, the second that of the technology the licensee already owns, which, unlike in the one-way case, affects how much it is willing to pay. Symmetrically, firm $i$ pays $L^x(\beta_j, \beta_i)$ for firm $j$'s technology. When no confusion arises, we write $L_i^x$ and $L_j^x$ for short. The lower bound (3) guarantees that the licensor is willing to grant the license; in the illustrative model of Section 6 the fee exceeds it at the equilibrium qualities. The two fees are paid in opposite directions, so that only the net transfer $L_i^x - L_j^x$ matters; at symmetric qualities it is zero.

Replacement never occurs when the two firms hold technologies of the same quality. It may, however, affect the incentive to invest more than the rival: a firm whose technology is substantially better induces the rival to replace its own, and can then sell it the whole technology rather than only the part the rival can add to its own. When this incentive is strong enough, the investment stage under non-overlapping trajectories no longer has a symmetric equilibrium; an asymmetric equilibrium emerges instead, characterized by one firm investing more than the other and by the latter replacing its own technology with the former's when both succeed. This happens only when complementarity is low: when innovations are highly complementary, triggering replacement requires a lead over the rival that is too costly to obtain. Formally, let $\theta_s$ denote the lowest degree of complementarity at which the investment stage under non-overlapping trajectories has a symmetric equilibrium, at which replacement does not occur. In Section 5, we show that $\theta_s > 0$ and discuss the asymmetric equilibrium that arises below it. In this section and the next, we focus on $\theta \in [\theta_s, 1]$. Since replacement requires $\beta_i < (1-\theta)\beta_j$, the effective quality under cross-licensing coincides with $\beta_i + \theta\beta_j$ at and around symmetric qualities, which suffices for our analysis. From now on and until Section 5, we therefore write it as $\beta_i + \theta\beta_j$.

### 3.3 Expected profits and investment

Expected profits weight the payoffs and licensing payments derived above by the probability of each R&D outcome. With probability $(1-p)^2$ both firms fail and earn $\pi_i(0,0)$. With probability $p(1-p)$ only firm $i$ succeeds and licenses its technology to firm $j$, and with the same probability the roles are reversed. With probability $p^2$ both firms succeed: under overlapping trajectories a single patent is granted, and each firm becomes the licensor or the licensee with probability $p^2/2$; under non-overlapping trajectories both patents are granted and the firms cross-license.

[11] Take-it-or-leave-it offers are not essential for cross-licensing. In the online Appendix C, the terms of each cross-license are set with equal bargaining power, under both fixed fees and royalties: the results are qualitatively unchanged, and the region in which the directional inefficiency survives is smaller (see footnote 27).

Expected profits in the two regimes are therefore

$$\Pi_i^o(\beta_i,\beta_j) = \left(\tfrac{p^2}{2}+p(1-p)\right)\Big[\tilde{\pi}_i(\beta_i,\beta_i)+L(\beta_i)+\tilde{\pi}_i(\beta_j,\beta_j)-L(\beta_j)\Big] + (1-p)^2\pi_i(0,0)-I(\beta_i), \tag{5}$$

$$\Pi_i^n(\beta_i,\beta_j) = p^2\Big[\tilde{\pi}_i^x(\beta_i+\theta\beta_j,\ \beta_j+\theta\beta_i)+L^x(\beta_i,\beta_j)-L^x(\beta_j,\beta_i)\Big] + p(1-p)\Big[\tilde{\pi}_i(\beta_i,\beta_i)+L(\beta_i)+\tilde{\pi}_i(\beta_j,\beta_j)-L(\beta_j)\Big]+(1-p)^2\pi_i(0,0)-I(\beta_i). \tag{6}$$

The two regimes differ only in the both-success event: a single patent, awarded to one of the two firms, under overlapping trajectories; a cross-licensing agreement between two distinct technologies under non-overlapping ones.

Let $\beta^o$ and $\beta^n$ denote the symmetric equilibrium investments in the two regimes, and $\Pi^{o*} \equiv \Pi^o(\beta^o,\beta^o)$ and $\Pi^{n*}(\theta) \equiv \Pi^n(\beta^n,\beta^n)$ the corresponding equilibrium expected profits. Only the latter depends on $\theta$: since cross-licensing takes place only along non-overlapping trajectories, the degree of complementarity between the two innovations affects firms' profits only in that regime.

**Regularity.** Throughout, we assume that, in each regime $d \in \{o,n\}$, expected profits are strictly concave in a firm's own investment, $\partial^2\Pi_i^d/\partial\beta_i^2 < 0$, and that the symmetric equilibrium is stable, $\partial^2\Pi_i^d/\partial\beta_i^2 + \partial^2\Pi_i^d/\partial\beta_i\partial\beta_j < 0$: a firm responds less than one-for-one to an increase in the rival's investment.[12]

One last condition, specific to the fixed-fee environment, disciplines marginal returns across the two regimes.

**Assumption 6.** *For any $\beta > 0$ and $\theta \in [\theta_s, 1]$:*

$$\frac{\partial}{\partial\beta_i}\Big[\tilde{\pi}_i^x(\beta_i+\theta\beta_j,\ \beta_j+\theta\beta_i)+L^x(\beta_i,\beta_j)-L^x(\beta_j,\beta_i)\Big]\Big|_{\beta_i=\beta_j=\beta} > \frac{1}{2}\frac{d}{d\beta}\Big[\tilde{\pi}(\beta,\beta)+L(\beta)\Big].$$

Assumption 6 is the natural extension of Assumption 2*b*) to an environment with licensing, and rests on the same trade-off between certainty and exclusivity.[13] As its counterpart in MS2025, it is a condition on marginal returns rather than on primitives of demand and competition. Consider the event in which both firms succeed. Under overlapping trajectories, a better innovation pays off only if the firm, rather than its rival, is awarded the patent, which happens half of the time; when it does, however, the firm is the sole owner of the technology and can monetize it both in the product market and by licensing it to the rival. Under non-overlapping trajectories, the firm owns its innovation for sure, but has to share the market with a rival that has innovated as well; it can still monetize a better technology through cross-licensing, since the rival is willing to pay more to access it. In MS2025, Assumption 2*b*) requires that certain

[12] Equivalently, along the symmetric locus the marginal return to investment crosses $I'(\cdot)$ once and from above. Both conditions hold when $I(\cdot)$ is sufficiently convex.

[13] Like all our assumptions, Assumption 6 is verified in the illustrative model of Section 6, for every $\theta \in [\theta_s, 1]$ and every admissible degree of product substitutability.

but shared ownership is worth more, at the margin, than a fifty-percent chance of exclusive ownership. Assumption 6 requires the same once licensing revenues are accounted for on both sides: the sure return to a better technology under differentiation should not be dominated by the uncertain return of winning the race and licensing it.

**Lemma 1.** *Under Assumptions 1–6, for any $\theta \in [\theta_s, 1]$, $\beta^n > \beta^o$: firms invest more along non-overlapping trajectories.*

The Lemma follows from Assumption 6: when both firms succeed, the sure return to a better technology under differentiation exceeds the uncertain return of being awarded the patent and licensing it, while the returns in the other events are the same in the two regimes. As in MS2025, firms therefore invest more along non-overlapping trajectories, and licensing strengthens rather than overturns this ranking.

## 3.4 The direction of innovation

The choice of trajectory is made comparing equilibrium expected profits.[14] We start from the benchmark of fully complementary innovations, and then turn to lower degrees of complementarity.

**Full complementarity.** When $\theta = 1$, each firm can add the rival's technology in full to its own: when both succeed, each operates the sum of the two technologies, $\beta_i + \beta_j$. Firms differentiate if $\Pi^{n*}(1) \geq \Pi^{o*}$. When (5) and (6) are evaluated at the equilibrium investments, all licensing payments cancel out, since investments are symmetric: under one-way licensing each firm is licensor and licensee with equal probability, and under cross-licensing the two fees are equal. Hence

$$\begin{aligned}\Pi^{o*} &= \big(p^2 + 2p(1-p)\big)\,\tilde{\pi}(\beta^o, \beta^o) + (1-p)^2\pi(0,0) - I(\beta^o),\\ \Pi^{n*}(1) &= p^2\,\tilde{\pi}^x(2\beta^n, 2\beta^n) + 2p(1-p)\,\tilde{\pi}(\beta^n, \beta^n) + (1-p)^2\pi(0,0) - I(\beta^n).\end{aligned}$$

The two expressions differ in two respects: the payoff when both firms succeed, $\tilde{\pi}$ under convergence and $\tilde{\pi}^x$ under differentiation, and the investment level, $\beta^o$ or $\beta^n$. To separate the two, we add and subtract $\Pi^n(\beta^o, \beta^o)$, the expected profit firms would earn on non-overlapping trajectories if they invested $\beta^o$. This term differs from $\Pi^{o*}$ only in the both-success event, so that

$$\Pi^{n*}(1) - \Pi^{o*} \;=\; \underbrace{p^2\Big[\tilde{\pi}^x(2\beta^o, 2\beta^o) - \tilde{\pi}(\beta^o, \beta^o)\Big]}_{\text{combination gain}} \;+\; \underbrace{\Big[\Pi^n(\beta^n, \beta^n) - \Pi^n(\beta^o, \beta^o)\Big]}_{\text{investment effect}}. \tag{7}$$

[14] With two symmetric trajectories, the first stage is a $2 \times 2$ game. If $\Pi^{n*} > \Pi^{o*}$, the game has two pure-strategy equilibria, in which firms are on different trajectories; if $\Pi^{o*} > \Pi^{n*}$, it has two pure-strategy equilibria, in which both firms are on the same trajectory. When indifferent, firms choose different trajectories. We disregard the equilibrium in mixed strategies, in which firms end up on the same trajectory with probability one half whatever the sign of $\Pi^{n*} - \Pi^{o*}$.

The first term compares the two directions at the same investment $\beta^o$. When both firms succeed, under differentiation each firm combines the two technologies and operates $2\beta^o$; under convergence a single patent is granted, and the two firms share one technology of quality $\beta^o$ through a license. We call this term the combination gain. In the appendix, we show that, by Assumptions 3$a$) and 4, this term is, in fact, a gain (i.e., it is positive). The second term, which we call the investment effect, compares the profits at the two investment levels along non-overlapping trajectories: it is the change in profits when firms raise their investment from $\beta^o$ to $\beta^n > \beta^o$ (Lemma 1). The investment effect is typically positive: since each firm operates both technologies, the rival's additional R&D also improves its own product. It may be negative only when product-market competition is so intense that the additional R&D partly dissipates profits.

Expression (7) leads to our first result.

**Proposition 1.** *With fully complementary innovations ($\theta = 1$), the combination gain is positive, and firms choose non-overlapping trajectories unless the additional investment induced by differentiation dissipates profits by more than the combination gain.*

With fully complementary innovations, cross-licensing turns the rival's success from a threat into an asset. By differentiating, firms that both succeed combine two technologies rather than share one, and the additional R&D that differentiation induces improves both products. Convergence can prevail only if product-market competition is so intense that this additional R&D erodes profits by more than the combination gain. In the illustrative model of Section 6, this never happens: at $\theta = 1$, firms differentiate for every degree of product substitutability.

**Partial complementarity.** By continuity, firms keep differentiating when $\theta$ is slightly below one. Everything else equal, a lower complementarity makes differentiation less attractive: when both firms succeed, each can absorb less of the rival's technology, and the cross-license adds less to what it already owns. The profits from convergence, instead, do not depend on $\theta$, since a single patent leaves nothing to combine. How low can complementarity then fall before firms prefer to converge? When $\theta$ changes, however, investment changes too, and this adds a second effect, which can go either way. Whether a single threshold exists therefore depends on how $\Pi^{n*}(\theta)$ changes with $\theta$ once investment adjusts: if it increases, there is a single threshold $\theta_F^*$, above which firms differentiate and below which they converge. Differentiating $\Pi^{n*}(\theta)$ with respect to $\theta$, and applying the envelope theorem to each firm's own investment choice, separates the two effects:

$$\frac{d\Pi^{n*}}{d\theta} = \underbrace{\left.\frac{\partial \Pi_i^n}{\partial \theta}\right|_{\beta_i=\beta_j=\beta^n}}_{\text{complementarity effect}} + \underbrace{\left.\frac{\partial \Pi_i^n}{\partial \beta_j}\right|_{\beta_i=\beta_j=\beta^n} \cdot \frac{d\beta^n}{d\theta}}_{\text{strategic effect}}. \tag{8}$$

The complementarity effect is the direct gain, at the current investment level, from a more valuable cross-license. From (6),

$$\left.\frac{\partial \Pi_i^n}{\partial \theta}\right|_{\beta_i=\beta_j=\beta^n} = p^2 \left[\frac{\partial \tilde{\pi}_i^x}{\partial \theta} + \frac{\partial \left(L_i^x - L_j^x\right)}{\partial \theta}\right] = p^2 \frac{\partial \tilde{\pi}_i^x}{\partial \theta} > 0,$$

where the inequality follows from Assumption 4. The fee term vanishes because, at symmetric qualities, a higher $\theta$ raises the fee each firm pays for the rival's technology and the fee it receives for its own by the same amount: the net transfer is zero for every $\theta$, and so is its derivative with respect to $\theta$ (Lemma 3 in the Appendix).

The strategic effect, instead, has an ambiguous sign. Under the regularity conditions of Section 3.3, $d\beta^n/d\theta > 0$ whenever a stronger complementarity raises the marginal return to a firm's own investment, $\partial^2\Pi_i^n/\partial\beta_i\partial\theta > 0$, as in the illustrative model of Section 6.[15] The sign of the strategic effect is then that of $\partial\Pi_i^n/\partial\beta_j$: whether a more capable rival helps or hurts firm $i$. A stronger rival is a tougher competitor and, when only the rival succeeds, it lowers what firm $i$ earns as a licensee; when both succeed, however, it typically pays more for firm $i$'s technology, since its better product sells more. Which force prevails depends on how intense product-market competition is and on how much of each technology the rival can absorb.

Expression (8) leads to our second result.

**Proposition 2.** *Suppose firms strictly prefer to differentiate at $\theta = 1$, $\Pi^{n*}(1) > \Pi^{o*}$. If the complementarity effect in (8) dominates the strategic effect, there exists a unique threshold $\theta_F^* \in [\theta_s, 1)$ such that firms choose non-overlapping trajectories if and only if $\theta \geq \theta_F^*$, with $\theta_F^* = \theta_s$ whenever $\Pi^{n*}(\theta_s) \geq \Pi^{o*}$.*

Provided that complementarity raises the marginal return to investment, a sufficient condition for the complementarity effect to dominate is that R&D costs are sufficiently convex (as in MS2025): a higher complementarity then changes investment little, and the direct gain from a more valuable cross-license prevails even when the strategic effect is negative. The condition need only hold for qualities above $\beta^o$, since $\beta^n > \beta^o$ by Lemma 1; the formal statement is in the Appendix. In the illustrative model of Section 6, it holds for every $\theta$ and degree of product substitutability.

Below $\theta_F^*$, firms choose the same trajectory. This is costly for consumers.

**Corollary 1.** *Under Assumptions 1–6, expected consumer surplus is higher under non-overlapping than under overlapping trajectories, at the respective equilibrium investment levels, for every $\theta \in [\theta_s, 1]$.*

This result follows directly from Lemma 1, that is, from the investment ranking $\beta^n > \beta^o$. It implies that whenever firms choose non-overlapping trajectories, their choice also maximizes total welfare, since consumer surplus and industry profits are both higher under differentiation.

Together, Propositions 1–2 and Corollary 1 qualify the main result of MS2025. In their setting, firms may converge on the same trajectory even though society would prefer them to differentiate: competing for a single patent gives the winner exclusive control of the technology,

[15] Differentiating the symmetric first-order condition $\partial\Pi_i^n(\beta_i, \beta^n; \theta)/\partial\beta_i = 0$ with respect to $\theta$ gives

$$\frac{d\beta^n}{d\theta} = -\frac{\partial^2\Pi_i^n/\partial\beta_i\partial\theta}{\partial^2\Pi_i^n/\partial\beta_i^2 + \partial^2\Pi_i^n/\partial\beta_i\partial\beta_j}.$$

The denominator is negative under the stability condition of Section 3.3, so $d\beta^n/d\theta$ has the sign of the numerator.

and this prospect can outweigh the certainty of owning a distinct but shared innovation. Cross-licensing changes this trade-off. Once firms can exchange their technologies, differentiation is no longer a choice between two separate and competing innovations, but also an opportunity to combine them, and the more complementary the innovations, the more valuable this opportunity becomes. When the conditions of Propositions 1 and 2 hold, as they do in the illustrative model of Section 6, the directional inefficiency of MS2025 disappears with fully complementary innovations, and can survive only for $\theta < \theta_F^*$, that is, when the innovations developed along the two trajectories are near-substitutes and the cross-license adds little to what each firm already owns. In this region, firms prefer to converge while consumers would be better off under differentiation; whether total welfare is also lower depends on the relative size of the two effects, and in the illustrative model of Section 6 differentiation is socially preferred throughout. How large the residual region is cannot be established in general; in the illustrative model the threshold $\theta_F^*$ is low, so that the inefficiency is confined to near-substitute innovations, where stricter patentability requirements remain the relevant instrument.

# 4 Royalty licensing

We now replace the lump-sum transfer with a per-unit royalty. As in Section 3, we focus on $\theta \in [\theta_s, 1]$, where the investment stage has a symmetric equilibrium at which replacement does not occur (the bound $\theta_s$ depends on the contractual form), and we write the effective quality under cross-licensing as $\beta_i + \theta\beta_j$. A licensee $j$ pays the licensor $i$ a royalty rate $r_i$ for every unit it sells of the final good improved with the licensed technology; the licensor $i$ collects $r_i$ on every such unit. Two features distinguish royalties from fees and drive all the results of this section. First, the royalty enters the licensee's marginal cost, thereby directly affecting product-market behavior. Second, the licensor's licensing income is proportional to the licensee's output, so the innovator keeps earning on the rival's sales. As under fixed fees, the patent holder makes a take-it-or-leave-it offer and sets the royalty at the highest rate the licensee accepts.

**One-way licensing.** When firm $i$ is the only patent holder, the licensee $j$ absorbs the full licensed technology $\beta_i$, and the licensor sets the royalty rate equal to the value it transfers, $r_i = \beta_i$ (and symmetrically $r_j = \beta_j$).[16] As under fixed fees, the licensee is left at its outside option.

[16] In the illustrative model of Section 6, as in any environment in which quality and the royalty enter the licensee's margin one-for-one, the transferred value is the highest rate the licensee accepts for non-drastic innovations, which are the objects of this paper: at that rate its margin, and hence its output, is exactly as without the license. The licensor, by contrast, strictly gains, since it collects the royalty on every unit sold by a rival that does not become a tougher competitor. In the illustrative model the rate the licensor would charge absent the licensee's participation constraint exceeds $\beta_i$ at the equilibrium investment levels, so that the constraint binds and $r_i = \beta_i$ is the outcome of a take-it-or-leave-it offer. The rule borrows from the well-established result that, for a non-drastic cost-reducing innovation of size $\varepsilon$, the optimal per-unit royalty equals $\varepsilon$, both for an outside innovator (Kamien and Tauman, 1986) and for an innovator that competes with its licensee (Wang, 1998).

**Cross-licensing.** When both firms hold a patent, consider, as in Section 3.2, the license on firm $i$'s technology, given that the reverse license is in place at rate $r_j^x$. Let $\overline{r}_i^x$ denote the highest rate firm $j$ is willing to pay, at which it is indifferent between taking the license and keeping its own technology only, and $\underline{r}_i^x$ the lowest rate firm $i$ is willing to accept, at which it is indifferent between granting the license and not granting it. As under fixed fees, each patent holder makes a take-it-or-leave-it offer for its own technology, taking the other license as given, so that

$$r_i^x = \overline{r}_i^x, \tag{9}$$

and symmetrically for firm $j$. The licensee is thus left at its outside option in each direction, and at symmetric qualities the two rates coincide.[17] Since each offer is made separately and is accepted only if it does not leave the licensee worse off than without that license, firms cannot jointly raise both rates above the value of the technologies. This rules out the mechanism studied by Jeon and Lefouili (2018), who show that unconstrained bilateral cross-licensing contracts allow firms to sustain the monopoly outcome.[18]

With royalties, gross payoffs must be evaluated at the (royalty-distorted) product-market equilibrium. Consistently with the two-argument notation of Section 2, we write:

- $\tilde{\pi}_i^{\ell}(\beta_i, \beta_i)$ when only $i$ succeeds and licenses to $j$ (the licensor payoff): both firms operate quality $\beta_i$, firm $j$ pays $r_i = \beta_i$ per unit, and firm $i$'s payoff is inclusive of the royalty income;
- $\tilde{\pi}_i^{f}(\beta_j, \beta_j)$ when only $j$ succeeds and $i$ takes the license (the licensee payoff): both firms operate quality $\beta_j$ and firm $i$ pays $r_j = \beta_j$ per unit;
- $\tilde{\pi}_i^{x}(\beta_i+\theta\beta_j,\ \beta_j+\theta\beta_i)$ when both succeed on non-overlapping trajectories and cross-license: firm $i$ operates $\beta_i + \theta\beta_j$, firm $j$ operates $\beta_j + \theta\beta_i$, and the royalty flows $r_i^x$ (received) and $r_j^x$ (paid) are exchanged, both included in $\tilde{\pi}_i^x$.

Exactly as under fixed fees, the complementarity parameter $\theta$ enters payoffs only through the cross-licensing event: under one-way licensing a single innovation exists, and the licensee absorbs its quality in full.

With the same state probabilities as in Section 3.3, expected profits become

$$\Pi_i^o(\beta_i, \beta_j) = \left(\tfrac{p^2}{2} + p(1-p)\right)\left[\tilde{\pi}_i^{\ell}(\beta_i, \beta_i) + \tilde{\pi}_i^{f}(\beta_j, \beta_j)\right] + (1-p)^2\pi_i(0,0) - I(\beta_i), \tag{10}$$

$$\begin{aligned}\Pi_i^n(\beta_i, \beta_j) = {} & p^2\,\tilde{\pi}_i^{x}(\beta_i + \theta\beta_j,\ \beta_j + \theta\beta_i) + p(1-p)\left[\tilde{\pi}_i^{\ell}(\beta_i, \beta_i) + \tilde{\pi}_i^{f}(\beta_j, \beta_j)\right] \\ & + (1-p)^2\pi_i(0,0) - I(\beta_i).\end{aligned} \tag{11}$$

As under fixed fees, the two regimes differ only in the both-success event. Two differences with respect to (5)–(6) are worth noting: the licensing payment no longer appears as a separate

[17] In the illustrative model of Section 6, $\overline{r}_i^x = \theta\beta_i$, the value of the technology transferred, and $\underline{r}_i^x < \overline{r}_i^x$, so that the licensor is willing to grant the license.

[18] Equivalently, we assume that the antitrust authority can detect and sanction such agreements, which are not the subject of this paper.

lump-sum term, since it is embedded in the royalty-distorted payoffs; and under cross-licensing there is no counterpart of the net fee $L_i^x - L_j^x$, since the two royalty flows are already accounted for in each firm's payoff and offset at symmetric investments. We define $\beta^o$, $\beta^n$, $\Pi^{o*}$ and $\Pi^{n*}(\theta)$ as in Section 3.3 and maintain the regularity conditions stated there. As under fixed fees, only $\Pi^{n*}$ depends on $\theta$.

We complement the assumptions of Section 2 with one condition specific to the royalty environment.[19]

**Assumption 7.** *For any $\beta > 0$ and $\theta \in [\theta_s, 1]$:*

*a)* $\tilde{\pi}^\ell(\beta, \beta) > \tilde{\pi}^f(\beta, \beta) \geq \pi(0, \beta)$;

*b)* $\tilde{\pi}^\ell(\beta, \beta)$ *increases in* $\beta$;

*c) at the margin,*

$$\left.\frac{\partial\, \tilde{\pi}_i^x(\beta_i + \theta\beta,\ \beta + \theta\beta_i)}{\partial \beta_i}\right|_{\beta_i = \beta} > \frac{1}{2}\,\frac{d\,\tilde{\pi}^\ell(\beta, \beta)}{d\beta};$$

*d) in the cross-licensing event,* $\overline{r}_i^x \leq \theta\beta_i$;

*e) with fully complementary innovations,* $\tilde{\pi}^x(2\beta, 2\beta) > \frac{1}{2}\left[\tilde{\pi}^\ell(\beta, \beta) + \tilde{\pi}^f(\beta, \beta)\right]$.

Part *a*) states that the licensor earns strictly more than the licensee, since it earns on its own product and also collects the royalty on every unit the rival sells; the licensee, in turn, weakly gains from taking the license, with equality when the royalty extracts its entire surplus, as it does under one-way licensing. Part *b*) states that a better technology strictly benefits the licensor. Part *c*) is the counterpart of Assumption 6 and rests on the same trade-off between certainty and exclusivity: it compares the sure return to a better technology under differentiation with half the return of a firm that is awarded the patent and licenses it. Under royalties, the former includes the royalty income $r_i^x$ the firm collects on each unit of the rival's output. Part *d*) states that the highest rate a licensee accepts does not exceed the value of the technology it receives. Together with take-it-or-leave-it offers, it ensures that technologies are not traded at a price above their value, which would amount to an anti-competitive use of cross-licensing. Part *e*) is the royalty counterpart of the combination gain in (7): combining two fully complementary technologies yields each firm more than the average of the licensor and licensee payoffs that a single technology, licensed through a royalty, would yield.

**Lemma 2.** *Under Assumptions 1–5 and 7, for any $\theta \in [\theta_s, 1]$, $\beta^n > \beta^o$: firms invest more along non-overlapping trajectories.*

The intuition is the same as under fixed fees. The two regimes differ only when both firms succeed. Under overlapping trajectories, a better innovation then pays off only if the firm, rather than its rival, is awarded the patent, which happens half of the time. Under non-overlapping trajectories, the firm keeps its patent for sure, and a better technology raises both

[19] As all our assumptions, Assumption 7 is verified in the illustrative model of Section 6.

the value of its own product and the royalty it collects on every unit the rival sells with it. Since the sure return to a better technology dominates the uncertain one (Assumption 7*c*)), firms invest more along non-overlapping trajectories, as in MS2025. Under royalties, the sure return has an additional component: under differentiation, the firm earns on its innovation twice, in the product market and through the rival's sales.

For the direction of innovation, we follow the same steps as under fixed fees, starting from fully complementary innovations. As in (7), the difference $\Pi^{n*}(1) - \Pi^{o*}$ splits into a combination gain, which compares the two directions at the investment $\beta^o$, and an investment effect, which compares the profits at $\beta^o$ and $\beta^n$ along non-overlapping trajectories. The investment effect has the same form as in (7). The combination gain, instead, becomes $p^2\big[\tilde{\pi}^x(2\beta^o, 2\beta^o) - \frac{1}{2}\big(\tilde{\pi}^\ell(\beta^o, \beta^o) + \tilde{\pi}^f(\beta^o, \beta^o)\big)\big]$, since under convergence the firm that wins the patent lottery earns the licensor's payoff and the other firm earns the licensee's payoff. It is positive by Assumption 7*e*), and Proposition 1 extends to royalty licensing (see the proof of Proposition 3).

For lower degrees of complementarity, the argument is again the same as under fixed fees: by continuity, firms keep differentiating when $\theta$ is slightly below one, and there is a single threshold if $\Pi^{n*}(\theta)$ increases with $\theta$. The derivative of $\Pi^{n*}$ decomposes as in (8). Since licensing payments are embedded in $\tilde{\pi}_i^x$, the complementarity effect is simply

$$\left.\frac{\partial \Pi_i^n}{\partial \theta}\right|_{\beta_i=\beta_j=\beta^n} = p^2 \left.\frac{\partial \tilde{\pi}_i^x}{\partial \theta}\right|_{\beta_i=\beta_j=\beta^n} > 0, \tag{12}$$

by Assumption 4. At symmetric qualities, a higher $\theta$ raises the royalty each firm pays for the rival's technology and the royalty it collects on its own by the same amount, so that the complementarity effect reflects only the higher value of the improved products. The strategic effect, again, has an ambiguous sign. Investment rises with complementarity, $d\beta^n/d\theta > 0$, whenever a higher $\theta$ raises the marginal return to a firm's own investment.[20] Under royalties this is less obvious than under fees: a higher $\theta$ raises the royalty a firm collects on each unit the rival sells, but a better own technology also reduces the rival's sales, on which the royalty is collected. In the illustrative model of Section 6 the first effect always prevails, and $\beta^n$ increases in $\theta$. The sign of the strategic effect is then that of $\partial \Pi_i^n/\partial \beta_j$, and the same forces as under fixed fees are at work, with the royalty a stronger rival pays on its larger output in place of the higher fee.

The threshold result of Proposition 2 extends to royalty licensing.

**Proposition 3.** *Suppose firms strictly prefer to differentiate at $\theta = 1$, $\Pi^{n*}(1) > \Pi^{o*}$. If the complementarity effect in (12) dominates the strategic effect, there exists a unique threshold $\theta_R^* \in [\theta_s, 1)$ such that firms choose non-overlapping trajectories if and only if $\theta \geq \theta_R^*$, with $\theta_R^* = \theta_s$ whenever $\Pi^{n*}(\theta_s) \geq \Pi^{o*}$.*

The sufficient condition discussed after Proposition 2 applies unchanged under royalties, and holds for every $\theta$ and degree of product substitutability in the illustrative model. A firm

[20] See footnote 15; the argument does not depend on the contractual form.

that innovates on its own trajectory and cross-licenses earns twice: on its product, and on every unit the rival sells with its technology. The stronger the complementarity, the more of each technology the rival absorbs and the larger the royalty flow, and the more attractive it is to pick distinct R&D territories. Convergence remains attractive only when there is little to gain from differentiation, that is, when $\theta$ is small; in that case, firms prefer to share a single technology licensed through a royalty, which keeps the licensee's output below what a lump-sum fee would allow.

Below $\theta_R^*$, firms choose the same trajectory. Again, this is costly for consumers.

**Corollary 2.** *Under Assumptions 1–5 and 7, expected consumer surplus, evaluated at the royalty-distorted equilibrium quantities, is higher under non-overlapping than under overlapping trajectories, at the respective equilibrium investment levels, for every* $\theta \in [\theta_s, 1]$.

As under fixed fees, the result follows from the investment ranking of Lemma 2. Two forces make differentiation better for consumers. First, firms invest more. Second, when both firms succeed each firm keeps its own innovation, whereas under convergence the royalty absorbs the value of the single technology the licensee receives (Assumption 5*b*). Complementarity, instead, adds nothing for consumers at given investments: its value is collected by licensors through the royalty rather than passed on through lower prices.

As under fixed fees, Corollary 2 implies that firms' private choice maximizes welfare whenever they choose non-overlapping trajectories; under the conditions of Propositions 1 and 3, royalty licensing removes the directional inefficiency of MS2025 with fully complementary innovations, and the inefficiency can survive only for $\theta < \theta_R^*$.

# 5 Near-substitute innovations and technology replacement

The analysis of Sections 3–4 rests on the symmetric investment equilibrium, which exists only for $\theta \geq \theta_s$. When innovations are close substitutes, this is no longer the case, and replacement shapes both investment and the direction of innovation.

As anticipated in Section 3.2, replacement changes the incentive to outspend the rival. If firm $i$ invests sufficiently more than firm $j$, $\beta_i > \beta_j/(1-\theta)$, firm $j$ prefers to replace its own technology and adopt $i$'s. The license then transfers $i$'s whole technology, rather than the part that $j$ can add to its own, and firm $i$ can charge for all of it: investing more than the rival pays off twice, through a better product and through the sale of the whole technology. Triggering replacement requires a lead of $\theta\beta_j/(1-\theta)$ over the rival. When complementarity is high, this lead is too costly to obtain (with $\theta = 1$, replacement never occurs), and the symmetric equilibrium survives; hence $\theta_s < 1$. When complementarity is low, a small lead suffices, and the symmetric equilibrium breaks down. The next result shows that this is always the case when innovations are perfect substitutes.

Under fixed fees, we assume, as in models of process innovation and in our illustration, that a license exactly duplicates the technology, $\tilde{\pi}(\beta, \beta) = \pi(\beta, \beta)$, and that joint-success profits

increase with the common quality of the two firms. Under royalties, instead, we maintain the assumption that quality and the per-unit royalty enter the licensee's margin one-for-one with a binding participation constraint.

**Proposition 4.** *With perfectly substitute innovations ($\theta = 0$), there is no symmetric investment equilibrium under non-overlapping trajectories, under either contract. Hence $\theta_s > 0$.*

The logic is simple. With perfectly substitute innovations, being slightly ahead of the rival flips who sells to whom. A firm whose technology is marginally better than the rival's sells its whole technology to a rival that discards its own; a firm whose technology is marginally worse buys the rival's technology and is left at its outside option. The return to investment therefore jumps upwards exactly at the rival's investment level, and no investment level can be a best response to itself.

For $\theta < \theta_s$, the investment stage under non-overlapping trajectories has asymmetric equilibria, in which one firm (the leader) invests more than the other (the follower); when both succeed, the follower replaces its own technology with the leader's. Differentiation is then no longer a choice between two combinable technologies: both firms still obtain a patent, but they compete over which of the two technologies both of them will eventually use. The firm that invests more becomes the licensor, while the other keeps its own patent, which serves as its outside option when it buys the rival's technology. Since firms are identical when they choose their trajectories, neither has a claim to the leader role. We therefore assume that, once firms are on non-overlapping trajectories, each of the two asymmetric equilibria is played with equal probability, as if a fair coin, observed by both firms before they invest, assigned the roles. Firms then choose non-overlapping trajectories only if the expected profits of this leader–follower configuration, the average of the leader's and the follower's, exceed those of overlapping trajectories, $\Pi^{o*}$, which replacement leaves unchanged. Whether they do depends on how intensely firms compete in the product market, so that the directional inefficiency of MS2025 may reappear for near-substitute innovations. In the illustrative model of Section 6, this happens only for products that are not too differentiated, and, at $p = 1/2$, $\theta_s$ never exceeds 0.19.

**Policy implications.** Sections 3–5 deliver a common message. Under both contractual forms, and under the conditions of Propositions 1–3, which hold in our illustration, the prospect of cross-licensing removes the directional inefficiency of MS2025 when innovations are fully complementary, and continues to prevent it as long as complementarity is high enough. Cross-licensing raises the value of differentiation and leaves that of convergence unchanged: when firms differentiate, two patents on distinct, complementary technologies can be combined; when they converge, a single patent is granted and can only be shared. The inefficiency can arise only for near-substitute innovations: then the cross-license adds little to what each firm already owns or, when replacement occurs, turns research along distinct trajectories into a contest over whose technology both firms will use.

Three policy implications follow. First, for patent policy: MS2025 identify patentability requirements as the instrument that corrects the directional failure; our analysis shows that, even under a generous patent regime, the prospect of cross-licensing can correct it when innovations are complementary. In our model licensing is frictionless; policies that reduce its transaction costs, such as clear intellectual property rights, standardized agreements and efficient dispute resolution, bring actual technology markets closer to this benchmark, and are therefore likely to complement stricter patentability requirements. Second, for antitrust: when firms trade technology via royalty rates, cross-licensing agreements are traditionally scrutinized with suspicion, as potential vehicles for collusion. Royalties, it is argued, raise marginal costs and can soften product-market competition. In our analysis, however, a firm never pays more for a technology than its value, which rules out the collusive agreements studied by Jeon and Lefouili (2018). Within this limit, royalties do not reduce output for given investments: in the illustrative model, the royalty exactly offsets the improvement it pays for, so that quantities are the same as without cross-licensing. The ex-post concern therefore applies to the form of the payment rather than to cross-licensing itself: compared with lump-sum fees, royalties prevent consumers from sharing in the gains from complementarity. At the same time, cross-licensing has an ex-ante benefit: by raising profits in the non-overlapping regime, it steers R&D toward technological differentiation, which is the socially preferred direction in our illustration. Since cross-licensing also changes investment and the direction of research, these observations do not establish, on their own, that permitting it raises welfare; they suggest caution before restricting an institution that can steer R&D in the socially preferred direction. Agreements that lead a firm to pay more for a technology than its value remain problematic, and should be scrutinized against the risk of collusive behavior in the product market. Third, the residual failure region indicates where each instrument is needed: in industries where alternative research paths lead to near-substitute innovations and products are close substitutes, such as alternative production processes for homogeneous goods or rival molecules for the same therapeutic indication, licensing may not restore efficient differentiation, and patentability requirements remain the relevant tool to address the directional inefficiency.

# 6 An illustrative model

We illustrate the general results in a linear Cournot duopoly with product innovation. Inverse demands are à la Singh–Vives,

$$p_i \;=\; a + \beta_i - q_i - \gamma q_j, \qquad \gamma \in (0,1],$$

where $\gamma$ measures product substitutability; both firms produce at constant marginal cost $c < a$, and an innovation of quality $\beta$ raises the innovator's demand intercept from $a$ to $a + \beta$. Under cross-licensing, a firm's demand intercept rises to $a + \hat{\beta}_i$, with $\hat{\beta}_i$ defined in (1). The two parameters capture distinct dimensions: $\gamma$ measures how close the two products are for consumers, $\theta$ how much of the rival's innovation a firm can add to its own. In the smartwatch

example of the Introduction, two smartwatches may be close substitutes for consumers and yet incorporate complementary innovations, such as a low-power display and a low-latency connectivity module (high $\gamma$, high $\theta$); two quite differentiated smartwatches may instead both extend battery life through different technologies (low $\gamma$, low $\theta$). R&D costs are $I(\beta) = \beta^2/2$. In what follows, we proceed with $a - c = 1$. This normalization is without loss of generality: equilibrium investments scale with $a - c$ and all payoffs, fees and welfare with $(a - c)^2$, so that signs, thresholds and failure regions are unaffected. The equilibrium output and gross payoff of a firm whose demand intercept, net of marginal cost and of any per-unit royalty it pays, is $1 + x$, facing a rival whose net intercept is $1 + y$, are

$$q(x, y) = \frac{2(1 + x) - \gamma(1 + y)}{4 - \gamma^2}, \qquad \pi(x, y) = q(x, y)^2. \tag{13}$$

Here $x$ and $y$ are the effective qualities of the two firms, net of royalties paid; royalty income received by a licensor is added to its payoff separately. Consumer surplus is $CS = (q_i^2 + q_j^2 + 2\gamma q_i q_j)/2$ at the equilibrium quantities.

All the assumptions of Sections 2–5 are verified. Assumptions 1, 2 and 3*b*) hold for every $\gamma \in (0, 1]$ provided the innovation is non-drastic, i.e., $\beta < (2 - \gamma)/\gamma$ — the only binding condition, which ensures $\pi(0, \beta) > 0$.[21] Since a fixed-fee license exactly duplicates the demand-enhancing innovation, both firms operate the same intercept $a+\beta$ and Assumptions 5*b*) and 3*a*) hold with equality: $\tilde{\pi}(\beta, \beta) = \pi(\beta, \beta)$ and $\widetilde{CS} = CS(\beta, \beta)$; moreover, $\pi(\beta, \beta) = [(1+\beta)/(2+\gamma)]^2$ increases in $\beta$, as required in Section 5. Finally, $\tilde{\pi}^x = \big[(1 + (1 + \theta)\beta)/(2 + \gamma)\big]^2$ and the corresponding $\widetilde{CS}^x$ are strictly increasing in $\theta$, so Assumptions 4 and 5*c*) hold for every $\gamma$ under fixed fees; we discuss royalties in Section 6.2.

**Leader–follower equilibria.** As shown in Section 5, when complementarity is low the possibility of replacement breaks down the symmetric investment equilibrium under non-overlapping trajectories. In the illustrative model, the bound $\theta_s$ below which this happens depends on $\gamma$ and $p$, and is derived in Appendix B; at $p = 1/2$, it ranges between 0.11 and 0.19. Below $\theta_s$ (except very close to it), the investment stage has two asymmetric equilibria, one for each assignment of the leader role; see Appendix B for their closed forms. As in Section 5, we evaluate $\Pi^{n*}$ as the expected profit of a firm that is equally likely to be the leader or the follower, and $W^{n*}$ at the leader–follower investment profile.[22]

[21] One-way licensing under fixed fees requires that licensing raises gross industry profits (Section 3.1): with the Cournot payoffs (13), $2\pi(\beta, \beta) - \pi(\beta, 0) - \pi(0, \beta) \propto \beta\left[2(2 - \gamma)^2 + \beta(\gamma^2 - 8\gamma + 4)\right]$, which is positive for every $\beta$ when $\gamma \leq 4 - 2\sqrt{3}$ and for $\beta < 2(2 - \gamma)^2/(8\gamma - \gamma^2 - 4)$ otherwise.

[22] We do not consider equilibria in mixed strategies where pure-strategy equilibria exist (see Appendix B for the narrow intervals in which they do not). If roles were known when trajectories are chosen, either firm could induce convergence by choosing the rival's trajectory, so that differentiation would require the follower's payoff, rather than the average, to exceed $\Pi^{o*}$. We verify that firms would then converge for low values of $\theta$ also when products are differentiated: under fixed fees, the threshold with known roles would be about 0.04 at $\gamma = 0.1$ and 0.12 at $\gamma = 0.3$, and would coincide with $\theta_s$ for intermediate values of $\gamma$. Its maximum would be unchanged, at 0.22 under fixed fees and 0.26 under royalties, so that the failure region would remain confined to near-substitute innovations, though no longer to products that are not too differentiated.

**Admissible parameters.** We say that a pair $(\gamma, \theta)$ is admissible, for a given $p$, when the equilibrium investments satisfy three conditions: the regularity conditions of Section 3.3; non-drastic effective qualities, including the quality $(1+\theta)\beta^n$ reached through cross-licensing, which must be below $(2-\gamma)/\gamma$; and, under fixed fees, the condition of Section 3.1 under which one-way licensing takes place. For $p \leq 1/2$, every $(\gamma, \theta) \in (0,1] \times [0,1]$ is admissible under both contracts. For higher values of $p$, investments are larger and some combinations become inadmissible, independently of replacement: stability fails for very differentiated products and highly complementary innovations (under fixed fees from $p \approx 3/4$, under royalties only for $p$ close to one), while non-drasticity and the feasibility of one-way licensing fail for close substitutes and highly complementary innovations. All figures are drawn at $p = 1/2$.

## 6.1 Fixed fees

With the Cournot payoffs (13) the one-way fee takes the simple form

$$L(\beta_i) = \frac{4\beta_i\left((2-\gamma)+(1-\gamma)\beta_i\right)}{(4-\gamma^2)^2},$$

and, when no replacement takes place, the cross-licensing fees, which follow from (4), are:

$$L^x(\beta_i, \beta_j) = L_i^x = \frac{4\beta_i\,\theta(2-\gamma-\beta_i\,(\gamma-\theta)+\beta_j\,(2-\gamma\theta))}{(4-\gamma^2)^2},$$

and symmetrically for $L_j^x$. The equilibrium investments under overlapping and non-overlapping trajectories are

$$\beta^o = \frac{(4-\gamma)(2-\gamma)(2-p)p}{(4-\gamma^2)^2-(8-(8-\gamma)\gamma)(2-p)p} \tag{14}$$

and, for $\theta \in [\theta_s, 1]$,

$$\beta^n = \frac{2(2-\gamma)p(4-\gamma-(2-\gamma)(1-\theta)p)}{(4-\gamma^2)^2-2p^2\left(6\gamma-2\gamma\theta(\theta+4)-(4+\gamma^2)\left(1-\theta(1+\theta)\right)\right)-2(8-(8-\gamma)\gamma)p}. \tag{15}$$

With fully complementary innovations, (15) simplifies to

$$\beta^n(1) = \frac{2p(4-\gamma)(2-\gamma)}{(4-\gamma^2)^2-2p\left(8-(8-\gamma)\gamma\right)-2p^2(2-\gamma)^2}.$$

One can see that $\beta^n$ increases in $\theta$, meaning that higher complementarity raises investments in the non-overlapping regime. The following Remarks are verified.

**Remark 1.** *Under fixed-fee licensing, $\beta^n > \beta^o$ for every $\gamma$ and $\theta \in [\theta_s, 1]$.*

**Remark 2.** *Under fixed-fee licensing and with fully complementary innovations, firms choose non-overlapping trajectories, and this choice maximizes expected welfare, for every $\gamma$.*[23]

[23] The differences $\Pi^{n*}(1) - \Pi^{o*}$ and $W^{n*}(1) - W^{o*}$ are, up to a positive factor, polynomials in $(\gamma, p)$ that are positive over the admissible domain.

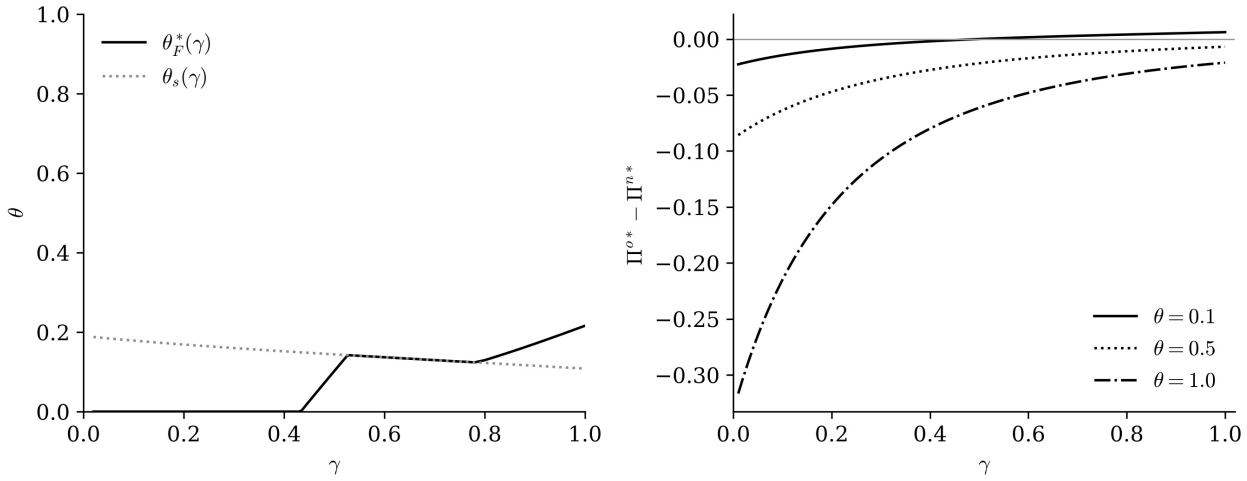



Figure 1: Fixed fees, $p = 1/2$, $a - c = 1$. Left: the threshold $\theta_F^*(\gamma)$ (solid) and the replacement bound $\theta_s(\gamma)$ (dotted). Right: $\Pi^{o*} - \Pi^{n*}$ as a function of $\gamma$ for $\theta \in \{0.1, 0.5, 1\}$.

*Proof.* Both Remarks follow from (14) and (15). □

Remark 1 is consistent with Lemma 1, and confirms one of the main results in MS2025: absent the threat of a patent race, firms invest more in R&D. Remark 2 illustrates Proposition 1: the combination gain in (7) is positive by construction, and the investment effect is positive as well for all but nearly homogeneous products. With fully complementary innovations, fixed-fee licensing removes the directional inefficiency of MS2025 for every degree of product substitutability.

**The direction of innovation.** Figure 1 (left) plots, at $p = 1/2$, the threshold $\theta_F^*(\gamma)$ above which firms choose non-overlapping trajectories, together with $\theta_s(\gamma)$, which decreases with $\gamma$. In the illustrative model, we define the threshold over the whole range of complementarity: $\theta_F^*$ is the lowest $\theta$ above which firms always choose non-overlapping trajectories, including in the leader–follower region $\theta < \theta_s$ (Appendix B). Unlike in Proposition 2, the threshold can therefore lie below $\theta_s$, which happens when firms differentiate in the leader–follower region as well.[24] The threshold is low throughout, never exceeding 0.22, and three ranges of product substitutability can be distinguished.[25] When products are sufficiently differentiated ($\gamma \lesssim 0.43$), firms differentiate for every $\theta$: below $\theta_s$, the leader–follower configuration is more profitable than convergence, because the leader's better technology reaches both products and competition erodes little of its value.[26] For intermediate degrees of substitutability ($0.43 \lesssim \gamma \lesssim 0.78$), the contest between leader and follower dissipates profits, through the leader's over-investment and the follower's duplicated effort, and firms differentiate only when complementarity is high

[24] Replacement is more attractive for the leader when products are differentiated: selling its whole technology to the rival then costs the leader little in the product market, since the rival's improved product takes little of its demand. As products become closer substitutes, this competitive cost rises, the incentive to outspend the rival weakens, and less complementarity is needed to sustain the symmetric equilibrium.

[25] For instance, $\theta_F^* \approx 0.10$ at $\gamma = 0.5$, $\approx 0.13$ at $\gamma = 0.7$, $\approx 0.17$ at $\gamma = 0.9$ and $\approx 0.22$ at $\gamma = 1$.

[26] At $\gamma = 0.5$ and $\theta = 0$, for instance, the leader invests 0.555 and the follower 0.387, against $\beta^o = 0.362$.

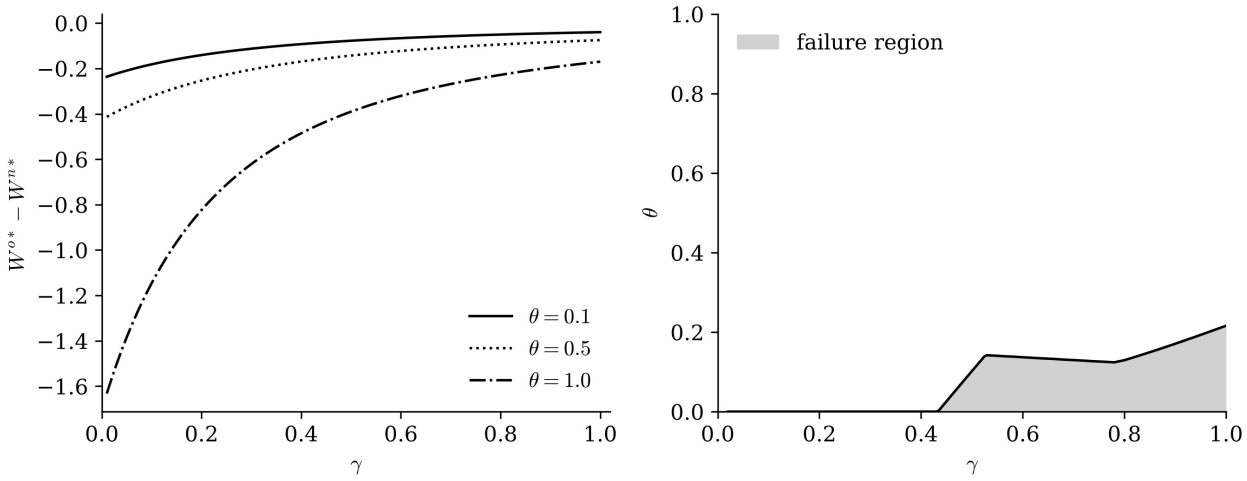



Figure 2: Fixed fees, $p = 1/2$, $a-c = 1$. Left: $W^{o*}-W^{n*}$ as a function of $\gamma$ for $\theta \in \{0.1, 0.5, 1\}$. Right: the market-failure region (gray), where firms choose overlapping trajectories while society prefers non-overlapping; the solid curve is $\theta_F^*(\gamma)$.

enough to restore the symmetric equilibrium: over most of this range the threshold coincides with $\theta_s$. Finally, when products are close substitutes ($\gamma \gtrsim 0.78$), the threshold lies above $\theta_s$ and is the one characterized in Proposition 2.[27] The right panel shows the profit differential $\Pi^{o*} - \Pi^{n*}$ underlying the threshold: it is negative everywhere except for low complementarity combined with $\gamma \gtrsim 0.49$, and it decreases in $\theta$ at every $\gamma$.[28]

**Welfare and the failure region.** Expected welfare ($W^d$, with $d \in \{o, n\}$) is computed as the aggregate sum of expected CS (see the proof of Corollary 1) and expected profits, with fees canceling out as transfers. Figure 2 (left) shows that $W^{o*}-W^{n*} < 0$ throughout: society prefers non-overlapping trajectories for every $(\gamma, \theta)$ [29] – even where firms privately choose to converge, and even below $\theta_s$, where the leader's technology reaches both products. The market-failure region – the gray area in the right panel, where $\Pi^{o*} > \Pi^{n*}$ and $W^{o*} < W^{n*}$ – is therefore the region below the threshold curve. It requires near-substitute innovations, $\theta < \theta_F^*(\gamma) \leq 0.22$, and products that are not too differentiated, $\gamma \gtrsim 0.43$.[30] Outside of it, fixed-fee licensing aligns

[27] The intensity of this contest depends on how the gain from replacement is shared. With take-it-or-leave-it offers, the leader obtains the follower's whole gain. If cross-licenses are instead negotiated with equal bargaining power, the leader obtains only half of it, both firms invest less, and firms differentiate for every $\theta$ up to $\gamma \approx 0.67$ under fixed fees and $\gamma \approx 0.58$ under royalties (online Appendix C). When products are close substitutes, the thresholds $\theta_F^*$ and $\theta_R^*$ are instead slightly higher than with take-it-or-leave-it offers.

[28] In the illustrative model, with $I''(\beta) = 1$, the convexity condition (ii) in the proof of Proposition 2 (Appendix) holds at every common quality $\beta \geq \beta^o$, for every $\gamma$ and $\theta$, under both contracts (indeed, for every $\beta$ above about one quarter of $\beta^o$). Condition (i), instead, fails unless products are sufficiently differentiated and innovations highly complementary; under fixed fees, at $\beta^n$ it fails for every $\theta$ when $\gamma \gtrsim 0.54$. When the rival can absorb little of the firm's technology, it pays little for it; when products are close substitutes, a stronger rival takes much of the firm's demand. In both cases, a stronger rival harms the firm.

[29] Except in the extremely narrow intervals close to $\theta_s$ in which the investment stage may have no equilibrium in pure strategies (see Appendix B for details): there, we do not characterize welfare.

[30] We verify that, without licensing, firms in the same Cournot economy converge only when products are close substitutes, $\gamma \gtrsim 0.87$, for every $\theta$. Licensing thus shrinks the failure region overall, but not uniformly: when innovations are near-substitutes and products are moderately differentiated, one-way licensing, which lets the loser of the patent lottery use the winner's technology, makes convergence more attractive than without

the private and the social direction of innovation.

## 6.2 Royalties

In the one-way event the licensee pays $r = \beta$ per unit for the full quality $\beta$: its effective margin falls back to $a - c = 1$, equilibrium quantities coincide with those of the corresponding no-licensing configuration, and

$$\tilde{\pi}^{\ell}(\beta, \beta) = \pi(\beta, 0) + \beta\, q(0, \beta), \qquad \tilde{\pi}^{f}(\beta, \beta) = \pi(0, \beta), \tag{16}$$

where $q(0, \beta)$ is the licensee's output: the licensor adds the royalty income to the payoff of a sole innovator, while the licensee is left exactly at its outside option.

In the cross-licensing event, the highest rate the licensee accepts takes a simple form. Taking the license on firm $i$'s technology at rate $r$ raises firm $j$'s demand intercept from $1+\beta_j$ to $1+\hat{\beta}_j$ and its marginal cost by $r$; the two effects cancel at $r = \hat{\beta}_j - \beta_j$, so that $\overline{r}_i^x = \hat{\beta}_j - \beta_j$, which equals $\theta\beta_i$ absent replacement and $\beta_i - \beta_j$ when firm $j$ replaces its own technology with $i$'s. With take-it-or-leave-it offers, $r_i^x = \overline{r}_i^x$ and, symmetrically, $r_j^x = \overline{r}_j^x$.

All the assumptions of Sections 2 and 4 are satisfied throughout the non-drastic region.[31]

The equilibrium investments under overlapping and non-overlapping trajectories are

$$\beta^o = \frac{(2-\gamma)\,(8-\gamma^2)\,(2-p)p}{2\left((4-\gamma^2)^2 - (4(1-\gamma)+\gamma^3)\,(2-p)p\right)} \tag{17}$$

and, for $\theta \in [\theta_s, 1]$,

$$\beta^n = \frac{(2-\gamma)p\,(8-\gamma^2-(4-\gamma^2)\,(1-\theta)p)}{(4-\gamma^2)^2 + 2p^2(-4\theta-\gamma(2-\gamma(\gamma(1-\theta)+\theta)-4\theta)) - 2\,(4(1-\gamma)+\gamma^3)\,p}, \tag{18}$$

which, with fully complementary innovations, simplifies to

$$\beta^n(1) = \frac{p(2-\gamma)(8-\gamma^2)}{(4-\gamma^2)^2 - 2p\,(4(1-\gamma)+\gamma^3) - 2p^2\,(4-2\gamma-\gamma^2)}.$$

Again, $\beta^n$ increases in $\theta$.

**Remark 3.** *Remarks 1 and 2 hold under royalty licensing.*

*Proof.* Follows from (17) and (18), as for Remarks 1 and 2. □

licensing. We also verify that expected welfare is higher with licensing than without for every $(\gamma, \theta)$.

[31] From (16), $\tilde{\pi}^{\ell} - \tilde{\pi}^{f} = \pi(\beta, 0) - \pi(0, \beta) + \beta\, q(0, \beta) > 0$ and $\tilde{\pi}^{f} = \pi(0, \beta)$: the licensee's participation constraint binds, so the second inequality in Assumption 7*a*) holds with equality. Part *b*) is immediate, since the licensor gains from $\beta$ both in the product market and through the royalty income. Part *d*) holds with equality. Moreover, at $r = \overline{r}_i^x$ the rival's margin is unchanged and firm $i$ strictly gains the royalty income, so that $\underline{r}_i^x < \overline{r}_i^x$ and the licensor is willing to grant the license. For part *e*), $\tilde{\pi}^x(2\beta, 2\beta) = \pi(\beta, \beta) + \beta\, q(\beta, \beta)$, which exceeds $\frac{1}{2}[\tilde{\pi}^{\ell}(\beta, \beta) + \tilde{\pi}^{f}(\beta, \beta)]$ for every $\gamma$ and every non-drastic $\beta$. Part *c*) and Assumptions 4 and 5*c*), which holds with equality, are verified numerically over a grid of admissible $(\gamma, \theta, \beta)$.

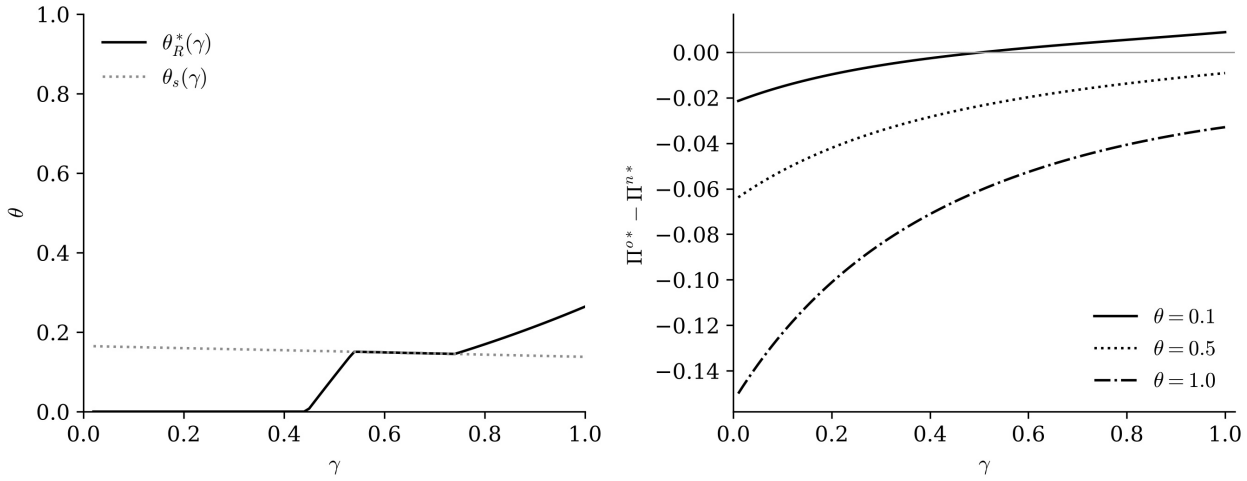



Figure 3: Royalties, $p = 1/2$, $a - c = 1$. Left: the threshold $\theta_R^*(\gamma)$ (solid) and the replacement bound $\theta_s(\gamma)$ (dotted). Right: $\Pi^{o*} - \Pi^{n*}$ as a function of $\gamma$ for $\theta \in \{0.1, 0.5, 1\}$.

At $p = 1/2$, the investment effect in (7) is positive for every $\gamma$ under royalties: with full complementarity, the additional R&D induced by differentiation raises firms' profits, which reinforces the combination gain.

**The direction of innovation.** Figure 3 (left) plots the threshold $\theta_R^*(\gamma)$ at $p = 1/2$, together with $\theta_s(\gamma)$. The three ranges identified under fixed fees reappear. When products are sufficiently differentiated ($\gamma \lesssim 0.44$), firms differentiate for every $\theta$. Below $\theta_s$, the leader earns on the follower's sales as well, collecting on every unit a royalty equal to the quality gap between the two technologies. For intermediate substitutability ($0.44 \lesssim \gamma \lesssim 0.74$) the threshold coincides with $\theta_s$ over most of the range, and for $\gamma \gtrsim 0.74$ it is the one of Proposition 3, which increases in product substitutability up to about 0.26.[32] The right panel shows the underlying profit differential $\Pi^{o*} - \Pi^{n*}$, which decreases in $\theta$ at every $\gamma$.

**Welfare and the failure region.** Expected welfare is computed as in Corollary 2: royalty payments cancel as transfers, but every surplus object is evaluated at the royalty-distorted quantities. Figure 4 (left) shows that $W^{o*} - W^{n*} < 0$ throughout: society prefers non-overlapping trajectories for every $(\gamma, \theta)$ (with the exception noted in footnote 29), even where firms privately choose to converge. The failure region (right panel) is the one below $\theta_R^*(\gamma)$, with the same qualitative shape as under fixed fees: with either contractual form, the directional inefficiency of MS2025 survives only when the innovations generated by the two trajectories are near-substitutes and products are not too differentiated.

## 6.3 Comparing the two contractual forms

Sections 3 and 4 analyze fixed fees and royalties separately, with payoffs in reduced form, so that the two contracts cannot be ranked in general. In the illustrative model, a direct ex-ante

[32] For instance, $\theta_R^* \approx 0.09$ at $\gamma = 0.5$, $\approx 0.15$ at $\gamma = 0.7$, $\approx 0.21$ at $\gamma = 0.9$ and $\approx 0.26$ at $\gamma = 1$.

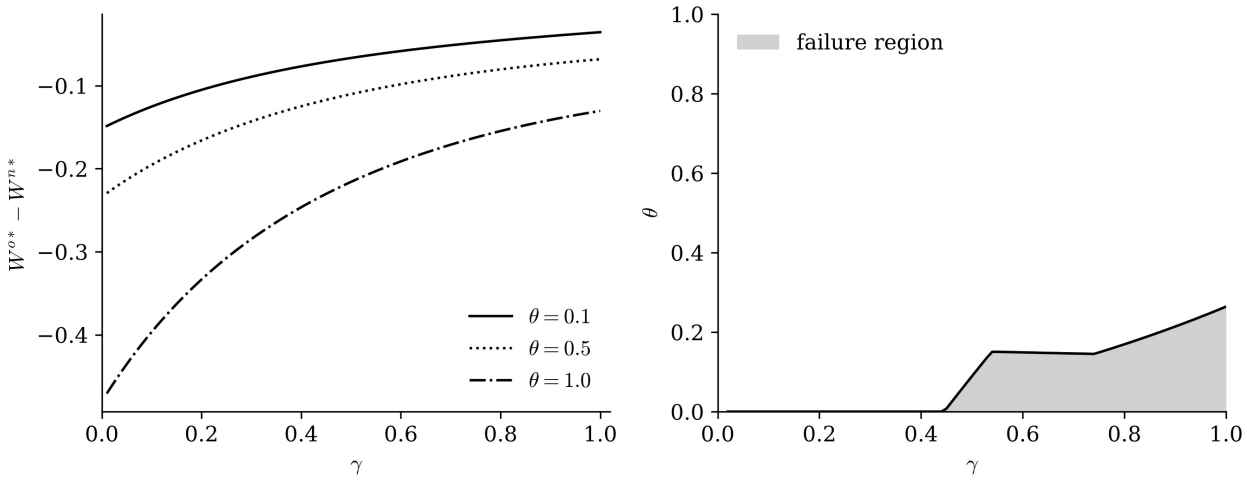



Figure 4: Royalties, $p = 1/2$, $a - c = 1$. Left: $W^{o*} - W^{n*}$ as a function of $\gamma$ for $\theta \in \{0.1, 0.5, 1\}$: always negative. Right: the market-failure region (gray), where firms choose overlapping trajectories while society prefers non-overlapping; the solid curve is $\theta_R^*(\gamma)$.

comparison is possible. Since under both contracts the patent holder makes take-it-or-leave-it offers, the comparison isolates the effect of the form of the payment. Figure 5 reports, as functions of $\gamma$, the two thresholds and the differences (fixed fee minus royalty) in equilibrium investment, profits and welfare, each evaluated in the regime firms choose; to compare the two contracts on symmetric equilibria, the last three panels consider values of $\theta$ above $\theta_s$ under both contracts.

Three findings emerge. First, on the direction, fixed fees align private and social incentives at least as well as royalties for almost every degree of product substitutability (Figure 5, top left): both thresholds are zero when products are sufficiently differentiated, and the royalty threshold is higher when products are close substitutes; only in a narrow window of intermediate substitutability, where both thresholds rise towards $\theta_s$, is the fixed-fee threshold slightly higher.[33] The reason is that a royalty raises the licensee's marginal cost and softens competition when firms converge and share a single technology, a force that rewards convergence and strengthens with product substitutability. For the same reason, firms earn higher equilibrium profits and invest more under royalties when products are sufficiently close substitutes: royalties pay more precisely where competition is intense and there is much to soften, a channel familiar from the literature on licensing between product-market rivals (Kamien and Tauman, 1986; Fan et al., 2018; Hsu et al., 2019).

Second, the welfare ranking is unambiguous at $p = 1/2$: fixed fees dominate royalties for every $\gamma$ and every $\theta$ in the figure. A per-unit royalty works as a tax internal to the industry. Under one-way licensing, the royalty leaves the licensee's output at its level without the license, while a lump-sum fee would raise it. Under cross-licensing, each royalty absorbs the value of the

[33] At $p = 1/2$, the royalty threshold is higher for $\gamma \gtrsim 0.53$, while for $\gamma \in (0.43, 0.53)$ the fixed-fee threshold is higher by at most 0.02. Profits are higher under royalties for $\gamma$ above a value between about 0.33 and 0.46, and investment for $\gamma$ above a value between about 0.54 and 0.69, depending on $\theta$, except in the window discussed below.

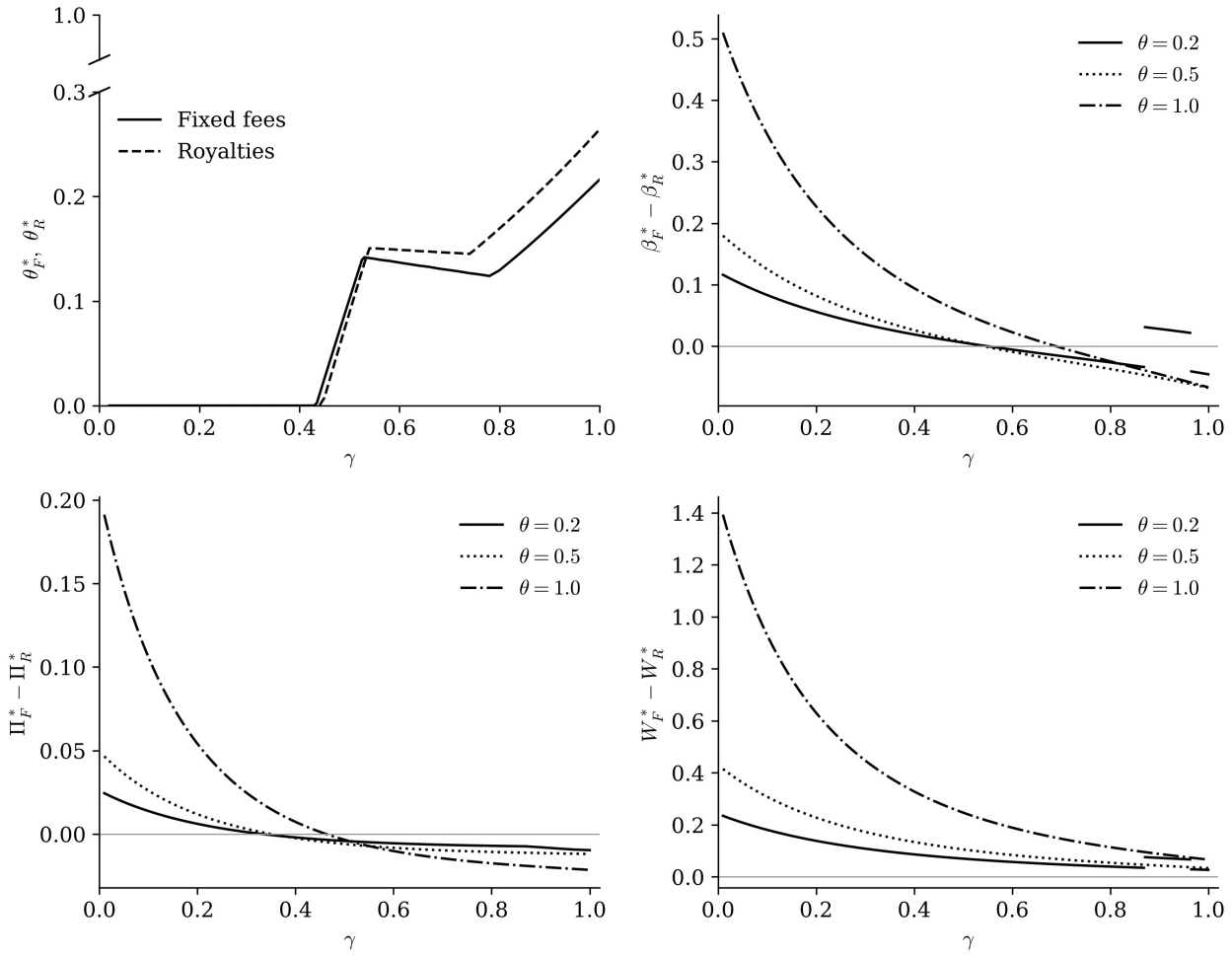



Figure 5: Fixed fees versus royalties ($a - c = 1$, $p = 1/2$). Top left: the thresholds $\theta_F^*(\gamma)$ and $\theta_R^*(\gamma)$ (note the break in the vertical axis). Top right: difference in equilibrium investment, $\beta_F^* - \beta_R^*$. Bottom left: difference in equilibrium profits, $\Pi_F^* - \Pi_R^*$. Bottom right: difference in equilibrium welfare, $W_F^* - W_R^*$, positive throughout. In the last three panels, $\theta \in \{0.2, 0.5, 1\}$.

technology the licensee receives, so that complementarity does not translate into lower prices, whereas lump-sum fees leave quantities undistorted. Investment reinforces this ranking when products are differentiated, where fixed fees also induce more R&D; when products are close substitutes royalties induce more R&D, but the output distortion prevails. The welfare gap is therefore largest where complementarity is strongest and products are most differentiated.

Third, the comparison isolates a region where the contract form determines the direction of innovation. For low complementarity ($\theta = 0.2$ in the figure) and $\gamma \in (0.87,\ 0.96)$, firms converge under royalties but still differentiate under fixed fees; the welfare gap widens discontinuously over this window, as the price distortion of the royalty is compounded by the directional failure. At the threshold, firms are indifferent between the two directions, so that moving to convergence costs them nothing, while it causes a discrete loss in welfare.

If firms could choose ex-ante the contractual form, the comparison of equilibrium profits indicates that they would prefer royalties when products are sufficiently close substitutes, that is, precisely where society prefers fees, including over the window in which royalties induce convergence. The choice of licensing instrument is therefore not innocuous, and, in contrast with analyses of the fee-versus-royalty question that focus on the intensity of innovation, the welfare ranking here is in favor of lump-sum arrangements.

# 7 Conclusions

This paper investigates whether the market for technology can correct the directional inefficiency of R&D that arises under a generous patent regime. Previous work shows that such a regime can induce firms to converge inefficiently on overlapping technological territories (Manenti and Sandrini, 2025); we show that the prospect of licensing mitigates this failure.

The corrective force comes from cross-licensing. One-way licensing takes place whatever the direction of research, whereas cross-licensing requires two patents on distinct technologies, and is therefore available only to firms that differentiate. When their innovations are complementary, cross-licensing allows firms to combine them, generating a surplus that convergence cannot offer. Under both fixed-fee and per-unit royalty contracts, this can induce firms to differentiate when innovations are sufficiently complementary, aligning their private incentives with society's preference for differentiation. In our illustration, the directional inefficiency survives only when innovations are near-substitutes and products are not too differentiated. There, either the cross-license adds little to what each firm already owns or, when replacement occurs, research along distinct trajectories turns into a contest over whose technology both firms will use.

Our findings suggest that technology markets and patentability requirements are complementary policy instruments: the prospect of cross-licensing aligns private and social incentives where innovations are complementary, while in industries prone to generating substitute innovations patentability requirements, the instrument identified by MS2025, remain the relevant tool. Our analysis also points to an ex-ante benefit of cross-licensing, which steers R&D to-

ward differentiation, and suggests caution before restricting the institution itself. As long as royalties do not exceed the value of the technologies exchanged, this benefit does not come at the cost of weaker competition: in the illustrative model, for given investments, royalties leave output where it would be without cross-licensing. The form of the contract, however, determines whether consumers share in the gains from complementarity: lump-sum fees pass them on and, in the illustrative model, dominate per-unit royalties in terms of welfare, even though firms would prefer royalties when products are close substitutes.

Our analysis abstracts from the transaction costs of licensing, which may prevent firms from exchanging their technologies; assessing how they affect the direction of innovation is left for future work.

# A Proofs

Throughout the appendix, equilibria of the investment stage are the symmetric solutions of the relevant first-order conditions, for $\theta \in [\theta_s, 1]$. Under the regularity conditions of Section 3.3, each marginal-return schedule crosses $I'(\cdot)$ once and from above. At symmetric qualities and for $\theta > 0$, replacement does not occur in a neighborhood of $\beta_i = \beta_j$, since it requires $\beta_i < (1-\theta)\beta_j$ or $\beta_j < (1-\theta)\beta_i$; hence $\hat{\beta}_i = \beta_i + \theta\beta_j$ locally, and payoffs are differentiable at symmetric points.

## A preliminary result

**Lemma 3.** *At symmetric qualities, $\beta_i = \beta_j$, the net cross-licensing transfer is zero for every $\theta \in [0,1]$: $L_i^x - L_j^x = 0$. Hence*

$$\left.\frac{\partial\big(L_i^x - L_j^x\big)}{\partial\theta}\right|_{\beta_i=\beta_j} = 0.$$

*Proof.* The counterfactuals defining $\overline{L}_i^x$ in (4) are mapped into those defining $\overline{L}_j^x$ by exchanging the roles of $i$ and $j$. At $\beta_i = \beta_j = \beta$ the exchange leaves every payoff unchanged, hence $L_i^x = \overline{L}_i^x = \overline{L}_j^x = L_j^x$. This holds for every $\theta$: at given symmetric qualities, the net transfer is identically zero as a function of $\theta$, and so is its derivative with respect to $\theta$. The argument does not extend to the derivative with respect to $\beta_i$, which moves qualities away from symmetry and need not be zero; this is why the term $L_i^x - L_j^x$ appears in the first-order condition (20). □

## Proof of Lemma 1

In (5), $\beta_j$ enters only through the terms $\tilde{\pi}_i(\beta_j, \beta_j) - L(\beta_j)$, which do not depend on $\beta_i$. Firm $i$'s first-order condition at a symmetric equilibrium is therefore

$$\left(\tfrac{p^2}{2} + p(1-p)\right)\frac{d}{d\beta}\Big[\tilde{\pi}(\beta,\beta) + L(\beta)\Big]\Big|_{\beta=\beta^o} \;=\; I'(\beta^o). \tag{19}$$

From (6),

$$p^2 \frac{\partial}{\partial \beta_i}\Big[\tilde{\pi}_i^x(\beta_i+\theta\beta_j,\beta_j+\theta\beta_i)+L^x(\beta_i,\beta_j)-L^x(\beta_j,\beta_i)\Big]\Big|_{\beta_i=\beta_j=\beta^n} + p(1-p)\,\frac{d}{d\beta}\Big[\tilde{\pi}(\beta,\beta)+L(\beta)\Big]\Big|_{\beta=\beta^n} = I'(\beta^n). \quad (20)$$

Subtracting the two marginal-return schedules at any common $\beta$, the terms for the event in which a single firm succeeds, which are the same in the two regimes, cancel out, and the difference reduces to the event in which both firms succeed:

$$p^2\left\{\frac{\partial}{\partial \beta_i}\Big[\tilde{\pi}_i^x(\beta_i+\theta\beta_j,\beta_j+\theta\beta_i)+L^x(\beta_i,\beta_j)-L^x(\beta_j,\beta_i)\Big]\Big|_{\beta_i=\beta_j=\beta} - \frac{1}{2}\frac{d}{d\beta}\Big[\tilde{\pi}(\beta,\beta)+L(\beta)\Big]\right\} > 0$$

where the inequality is Assumption 6. The marginal return under non-overlapping trajectories therefore exceeds that under overlapping trajectories pointwise. Since each schedule crosses $I'(\cdot)$ once and from above, the equilibrium investment defined by (20) lies strictly to the right of that defined by (19): $\beta^n > \beta^o$. The argument holds for every $\theta \in [\theta_s, 1]$, as Assumption 6 does. □

## Proof of Proposition 1

At $\theta = 1$, $\hat{\beta}_i = \beta_i + \beta_j$ for every pair of qualities, so that no replacement occurs and the regularity conditions guarantee that $\beta^n$ is an equilibrium. By Assumption 4, $\tilde{\pi}^x(2\beta, 2\beta) > \tilde{\pi}^x|_{\theta=0} = \pi(\beta,\beta)$, and by Assumption 3*a*), $\pi(\beta,\beta) \geq \tilde{\pi}(\beta,\beta)$, for every $\beta > 0$. At $\beta = \beta^o$, the combination gain in (7) is therefore strictly positive, and firms differentiate unless the investment effect is negative and larger, in absolute value, than the combination gain.

The investment effect is positive, for instance, when a stronger rival does not harm a firm. Let $V(\beta) \equiv \Pi^n(\beta,\beta)$ denote expected profits along the symmetric locus, so that $V'(\beta) = \partial\Pi_i^n/\partial\beta_i + \partial\Pi_i^n/\partial\beta_j$, both evaluated at $\beta_i = \beta_j = \beta$. By the regularity conditions, the marginal return to own investment along the symmetric locus crosses $I'(\cdot)$ once and from above at $\beta^n$, so that $\partial\Pi_i^n/\partial\beta_i > 0$ for every $\beta < \beta^n$. If $\partial\Pi_i^n/\partial\beta_j \geq 0$, then $V'(\beta) > 0$ on $[\beta^o, \beta^n)$, and since $\beta^n > \beta^o$ (Lemma 1) the investment effect $V(\beta^n) - V(\beta^o)$ is positive. Both terms in (7) are then positive, and $\Pi^{n*}(1) > \Pi^{o*}$. It suffices that $\partial\Pi_i^n/\partial\beta_j \geq 0$ holds at every common $\beta \in [\beta^o, \beta^n]$. □

## Proof of Proposition 2

The complementarity parameter $\theta$ enters expected profits only through the both-success, non-overlapping event, that is, through the payoff $\tilde{\pi}_i^x$ and the fees $L_i^x, L_j^x$, which do not appear in $\Pi_i^o$ (equation (5)). The objective firms maximize under overlapping trajectories is therefore independent of $\theta$, and so are $\beta^o$ and $\Pi^{o*}$.

Under the regularity conditions of Section 3.3, $\beta^n(\theta)$ is continuous in $\theta$, and so is $\Pi^{n*}(\theta)$; moreover, $\Pi^{n*}(1) > \Pi^{o*}$ by assumption. If the complementarity effect dominates the strategic

effect, $\Pi^{n*}$ is also strictly increasing. If $\Pi^{n*}(\theta_s) \geq \Pi^{o*}$, firms differentiate for every $\theta \in [\theta_s, 1]$, and $\theta_F^* = \theta_s$. Otherwise, $\Pi^{n*}(\theta_s) < \Pi^{o*} < \Pi^{n*}(1)$, and the intermediate value theorem and strict monotonicity deliver a unique $\theta_F^* \in (\theta_s, 1)$ such that $\Pi^{n*}(\theta) \geq \Pi^{o*}$ if and only if $\theta \geq \theta_F^*$. Even without monotonicity, continuity and $\Pi^{n*}(1) > \Pi^{o*}$ imply that firms differentiate for every $\theta$ in a neighborhood of 1.

Sufficient conditions. Assume $\partial^2 \Pi_i^n / \partial \beta_i \partial \theta > 0$. The complementarity effect dominates the strategic effect if, for every $\theta \in [\theta_s, 1]$ and every $\beta \geq \beta^o$, with all derivatives evaluated at $\beta_i = \beta_j = \beta$, either (i) $\partial \Pi_i^n / \partial \beta_j \geq 0$, or (ii)

$$-\Big(\frac{\partial^2 \Pi_i^n}{\partial \beta_i^2} + \frac{\partial^2 \Pi_i^n}{\partial \beta_i \partial \beta_j}\Big) \;>\; \max\Big\{0, -\frac{\partial \Pi_i^n}{\partial \beta_j}\Big\} \frac{\partial^2 \Pi_i^n / \partial \beta_i \partial \theta}{\partial \Pi_i^n / \partial \theta}.$$

Since $\partial^2 \Pi_i^n / \partial \beta_i^2$ includes $-I''(\beta)$, condition (ii) holds when R&D costs are sufficiently convex. To see this, note that by (8) and footnote 15, at $\beta = \beta^n(\theta)$,

$$\frac{d\Pi^{n*}}{d\theta} = \frac{\partial \Pi_i^n}{\partial \theta} - \frac{\partial \Pi_i^n}{\partial \beta_j} \frac{\partial^2 \Pi_i^n / \partial \beta_i \partial \theta}{\partial^2 \Pi_i^n / \partial \beta_i^2 + \partial^2 \Pi_i^n / \partial \beta_i \partial \beta_j},$$

where the denominator is negative by the stability condition of Section 3.3, and $\partial \Pi_i^n / \partial \theta = p^2 \, \partial \tilde{\pi}_i^x / \partial \theta > 0$ by Assumption 4 and Lemma 3. Under condition (i), both terms are non-negative and the first is strictly positive. Under condition (ii), rearranging gives $d\Pi^{n*}/d\theta > 0$ directly. Since $\beta^n(\theta) > \beta^o$ for every $\theta \in [\theta_s, 1]$ (Lemma 1), it suffices that either condition holds for $\beta \geq \beta^o$. □

## Proof of Corollary 1

Expected consumer surplus at a symmetric investment $\beta$ is

$$\begin{aligned} CS^o(\beta, \beta) &= p(2-p)\, \widetilde{CS}(\beta, \beta) + (1-p)^2 CS(0,0), \\ CS^n(\beta, \beta) &= p^2\, \widetilde{CS}^x\big((1+\theta)\beta, (1+\theta)\beta\big) + 2p(1-p)\, \widetilde{CS}(\beta, \beta) + (1-p)^2 CS(0,0). \end{aligned}$$

By the first part of Assumption 5*a*), the two partial derivatives of $CS$ are equal and positive, so their sum—the total derivative along the diagonal—is positive: $CS(\beta, \beta)$ is strictly increasing in $\beta$. By the second part of Assumption 5*a*), so is $\widetilde{CS}(\beta, \beta)$. Since $p(2-p) = p^2 + 2p(1-p)$,

$$\begin{aligned} CS^n(\beta^n, \beta^n) - CS^o(\beta^o, \beta^o) = p^2 \Big[\widetilde{CS}^x\big((1+\theta)\beta^n, (1+\theta)\beta^n\big) - \widetilde{CS}(\beta^o, \beta^o)\Big] \\ + 2p(1-p)\Big[\widetilde{CS}(\beta^n, \beta^n) - \widetilde{CS}(\beta^o, \beta^o)\Big]. \end{aligned}$$

Since $\beta^n > \beta^o$ (Lemma 1), the second bracket is positive by the monotonicity just established. For the first, Assumption 5*c*) gives $\widetilde{CS}^x\big((1+\theta)\beta^n, (1+\theta)\beta^n\big) \geq CS(\beta^n, \beta^n)$; monotonicity of $CS(\beta, \beta)$ together with $\beta^n > \beta^o$ gives $CS(\beta^n, \beta^n) > CS(\beta^o, \beta^o)$; and Assumption 5*b*) gives $CS(\beta^o, \beta^o) \geq \widetilde{CS}(\beta^o, \beta^o)$. Chaining the three, the first bracket is positive too, and $CS^n(\beta^n, \beta^n) > CS^o(\beta^o, \beta^o)$. When firms differentiate, $\Pi^{n*} \geq \Pi^{o*}$, so that industry profits are

also higher, and so is total welfare. □

## Proof of Lemma 2

In (10), $\beta_j$ enters only through the licensee term $\tilde{\pi}_i^f(\beta_j, \beta_j)$, which does not depend on $\beta_i$. Firm $i$'s first-order condition at a symmetric equilibrium is therefore

$$\left(\tfrac{p^2}{2} + p(1-p)\right) \frac{d\,\tilde{\pi}^\ell(\beta,\beta)}{d\beta}\bigg|_{\beta=\beta^o} = I'(\beta^o).$$

Moreover, from (11), we have

$$p^2 \frac{\partial\,\tilde{\pi}_i^x}{\partial\beta_i}\bigg|_{\beta_i=\beta_j=\beta^n} + p(1-p)\,\frac{d\,\tilde{\pi}^\ell(\beta,\beta)}{d\beta}\bigg|_{\beta=\beta^n} = I'(\beta^n),$$

where all licensing income is embedded in the payoffs—no separate transfer term appears, and $\partial\tilde{\pi}_i^x/\partial\beta_i$ includes the effect of $\beta_i$ on the royalties exchanged under cross-licensing. Subtracting the two marginal-return schedules at any common $\beta$, the terms for the event in which a single firm succeeds, which are the same in the two regimes, cancel out, and the difference reduces to the event in which both firms succeed:

$$p^2 \left\{ \frac{\partial\,\tilde{\pi}_i^x}{\partial\beta_i}\bigg|_{\beta_i=\beta_j=\beta} - \frac{1}{2}\,\frac{d\,\tilde{\pi}^\ell(\beta,\beta)}{d\beta} \right\} > 0$$

by Assumption 7*c*). As each marginal-return schedule crosses $I'(\cdot)$ once and from above, $\beta^n > \beta^o$. □

## Proof of Proposition 3

We first show that Proposition 1 extends to royalty licensing, as stated in Section 4. At a common investment $\beta$ and $\theta = 1$, (10) and (11) give $\Pi^n(\beta,\beta) - \Pi^o(\beta,\beta) = p^2\big[\tilde{\pi}^x(2\beta,2\beta) - \frac{1}{2}(\tilde{\pi}^\ell(\beta,\beta) + \tilde{\pi}^f(\beta,\beta))\big]$, which is positive by Assumption 7*e*). The rest of the argument for Proposition 1 carries over, with Lemma 2 in place of Lemma 1. For Proposition 3, the one-way rate $r = \beta$ does not depend on $\theta$, so that $\theta$ enters expected profits only through the cross-licensing event, which does not appear in (10); hence $\beta^o$ and $\Pi^{o*}$ are independent of $\theta$. From (11), $\partial\Pi_i^n/\partial\theta = p^2\,\partial\tilde{\pi}_i^x/\partial\theta$, which is positive at symmetric qualities by Assumption 4. The rest of the argument is identical, with Lemma 2 in place of Lemma 1. □

## Proof of Corollary 2

Royalty payments are transfers between the two firms and do not enter consumer surplus. Unlike lump-sum fees, however, the running royalty distorts product-market behavior, so that $\widetilde{CS}$ and $\widetilde{CS}^x$ are evaluated at the royalty-distorted equilibrium quantities, and Assumption 5 applies to these objects (footnote 8). With this interpretation, the expressions for expected

consumer surplus in the two regimes are those in the proof of Corollary 1, and the argument is identical, with Lemma 2 in place of Lemma 1. □

### Proof of Proposition 4

Let $\theta = 0$, so that $\hat{\beta}_i = \max\{\beta_i, \beta_j\}$, and fix the rival's investment at $\beta > 0$. Only the both-success term of (6) (respectively, (11)) depends on whether $\beta_i$ is above or below $\beta$; the one-way terms and $I(\beta_i)$ are differentiable in $\beta_i$. Consider the both-success event, and let $\partial_1\pi(\beta,\beta)$ and $\partial_2\pi(\beta,\beta)$ denote the partial derivatives of $\pi$ with respect to its first and second argument, evaluated at $(\beta,\beta)$, so that $d\pi(\beta,\beta)/d\beta = \partial_1\pi(\beta,\beta) + \partial_2\pi(\beta,\beta)$.

Fixed fees. If $\beta_i < \beta$, firm $i$'s technology is worthless to $j$, and $i$ replaces its own technology with $j$'s. With $\tilde{\pi} = \pi$, the license is signed if $2\pi(\beta,\beta) - \pi(\beta,\beta_i) - \pi(\beta_i,\beta) \geq 0$: this difference is zero at $\beta_i = \beta$ and has derivative $-d\pi(\beta,\beta)/d\beta < 0$ there, so it is positive for $\beta_i$ slightly below $\beta$. Under the take-it-or-leave-it offer, $i$ pays its whole gain and is left with its outside option, $\pi(\beta_i,\beta)$, whose derivative at $\beta_i = \beta$ is $\partial_1\pi(\beta,\beta)$. If $\beta_i > \beta$, the roles are reversed: by the same argument, the license is signed for $\beta_i$ slightly above $\beta$; $j$ replaces its technology with $i$'s, both firms operate $\beta_i$, and $i$ collects $j$'s gain, $\pi(\beta_i,\beta_i) - \pi(\beta,\beta_i)$. Firm $i$'s payoff is $2\pi(\beta_i,\beta_i) - \pi(\beta,\beta_i)$, whose derivative at $\beta_i = \beta$ is $2\partial_1\pi(\beta,\beta) + \partial_2\pi(\beta,\beta)$. The right derivative of $\Pi_i^n(\cdot,\beta)$ at $\beta_i = \beta$ therefore exceeds the left derivative by $p^2\, d\pi(\beta,\beta)/d\beta > 0$.

Royalties. Let $q(x,y)$ denote the equilibrium output of a firm operating quality $x$ against a rival operating $y$. If $\beta_i > \beta$, the rival replaces its technology with $i$'s at the rate $\overline{r}_i^x = \beta_i - \beta$; at this rate the rival's margin is unchanged, so that quantities are those of the configuration in which the two firms operate $\beta_i$ and $\beta$ without licensing, and firm $i$ earns $\pi(\beta_i,\beta) + (\beta_i - \beta)\, q(\beta,\beta_i)$. If $\beta_i < \beta$, firm $i$ replaces its technology with $j$'s at the rate $\beta - \beta_i$, its margin is unchanged, and it earns $\pi(\beta_i,\beta)$. The right derivative at $\beta_i = \beta$ therefore exceeds the left derivative by $p^2 q(\beta,\beta) > 0$.

Under either contract, a best response at $\beta_i = \beta$ would require the left derivative to be non-negative and the right derivative to be non-positive, which is impossible. Hence no symmetric profile with positive investment is an equilibrium at $\theta = 0$; nor is the profile with zero investment, at which, with $I'(0) = 0$, the marginal return to investment is positive. As $\theta \to 0$, the symmetric candidate $\beta^n(\theta)$ converges to a positive level, and expected profits are continuous in investments and $\theta$; the profitable deviation at $\theta = 0$ therefore remains profitable for $\theta$ close to zero, and $\theta_s > 0$. □

## B The illustrative model: technology replacement

This appendix characterizes, in the Cournot model of Section 6, the replacement bound $\theta_s$ and the investment equilibrium below it. Throughout, $q(x,y)$ and $\pi(x,y) = q(x,y)^2$ are the output and gross payoff in (13).

**Replacement deviations.** Let $\beta^n$ be the symmetric candidate, given by (15) under fixed fees and by (18) under royalties, and suppose firm $i$ deviates to $x > \beta^n/(1-\theta)$. When both firms succeed, the rival replaces its own technology with $i$'s, while $i$ adds the share $\theta\beta^n$ of the rival's technology to its own. Under fixed fees, firm $i$ pays for the rival's technology its whole gain, and is thus left with $\pi(x,x)$; it also collects the rival's whole gain from replacement, $\pi(x, x+\theta\beta^n) - \pi(\beta^n, x+\theta\beta^n)$. Under royalties, the rival pays $x-\beta^n$ per unit and firm $i$ pays $\theta\beta^n$ per unit, so that both margins are unchanged, and firm $i$ earns $\pi(x,\beta^n)+(x-\beta^n)\,q(\beta^n,x)$. Adding the other R&D outcomes, and assuming that licensing takes place at the deviation qualities (footnote 34), the expected profits from the deviation are

$$\begin{aligned}
\Pi_F^{\text{dev}}(x) &= p^2\big[\pi(x,x) + \pi(x, x+\theta\beta^n) - \pi(\beta^n, x+\theta\beta^n)\big] \\
&\quad + p(1-p)\big[2\pi(x,x) - \pi(0,x) + \pi(0,\beta^n)\big] + (1-p)^2\pi(0,0) - x^2/2, \\
\Pi_R^{\text{dev}}(x) &= p^2\big[\pi(x,\beta^n) + (x-\beta^n)\,q(\beta^n,x)\big] + p(1-p)\big[\pi(x,0) + x\,q(0,x) + \pi(0,\beta^n)\big] \\
&\quad + (1-p)^2\pi(0,0) - x^2/2.
\end{aligned}$$

Both are quadratic functions of $x$, concave for every $\gamma \in (0,1]$ and $p \in (0,1]$, and maximized at

$$x_F^* = \frac{2p\big[(2-\gamma)(4-\gamma) + 2\gamma p(1-\theta)\beta^n\big]}{(4-\gamma^2)^2 - 2p\,(8-8\gamma+\gamma^2)}, \qquad x_R^* = \frac{p\big[(2-\gamma)(8-\gamma^2) + p\,(8-2\gamma^2-\gamma^3)\beta^n\big]}{(4-\gamma^2)^2 - 2p\,(4-4\gamma+\gamma^3)}.$$

**The bound $\theta_s$.** The symmetric candidate is an equilibrium if and only if no replacement deviation is profitable, that is, if and only if $\Pi^{dev}(x^*) \le \Pi^n(\beta^n,\beta^n)$. The bound $\theta_s$ is the value of $\theta$ at which this condition holds with equality. For every value of $p$ and $\gamma$ we examined, under both contracts, the equation has a unique solution in $(0,1)$, at which $x^* > \beta^n/(1-\theta)$, so that the optimal deviation does trigger replacement.[34] We solve it numerically. The bound $\theta_s$ increases with $p$: replacement matters only when both firms succeed, an event whose probability rises with $p$. At $p = 1/2$, $\theta_s$ ranges between 0.11 and 0.19.

**The leader–follower equilibrium.** For $\theta < \theta_s$, let $\beta_H$ and $\beta_L < (1-\theta)\beta_H$ denote the investments of the leader and the follower. The leader's expected profits are those of the deviation above, with $\beta_L$ in place of $\beta^n$, so that its best response is $x^*(\beta_L)$. When both firms succeed, the follower replaces its own technology with the leader's. Under fixed fees, it is left with what it earns with its own technology against the leader's improved one, $\pi(\beta_L, \beta_H + \theta\beta_L)$, and collects the leader's whole gain from its technology, $\pi(\beta_H + \theta\beta_L, \beta_H) - \pi(\beta_H, \beta_H)$. Under royalties, the follower pays $\beta_H - \beta_L$ per unit for the leader's technology and the leader pays $\theta\beta_L$ per unit for the share it adds, so that both margins are unchanged; the follower earns

[34] We verify, at $p = 1/2$ and for a set of values of $\gamma$ and $\theta \in [\theta_s, 1]$, that deviations in the opposite direction, to an investment low enough that the firm replaces its own technology with the rival's, $x < (1-\theta)\beta^n$, are never profitable. We also verify that no deviation is profitable over the whole range of investments up to $\beta = 3$, with continuation payoffs that account for every case: drastic innovations, where the Cournot equilibrium is at a corner, and, under fixed fees, qualities at which licensing does not raise joint profits and therefore does not take place.

$\pi(\beta_L, \beta_H)$ and collects $\theta\beta_L$ on each unit the leader sells. Adding the other R&D outcomes, the follower's expected profits are

$$\begin{aligned}\Pi_F^L(\beta_L) &= p^2\big[\pi(\beta_L, \beta_H + \theta\beta_L) + \pi(\beta_H + \theta\beta_L, \beta_H) - \pi(\beta_H, \beta_H)\big] \\ &\quad + p(1-p)\big[2\pi(\beta_L, \beta_L) - \pi(0, \beta_L) + \pi(0, \beta_H)\big] + (1-p)^2\pi(0,0) - \beta_L^2/2, \\ \Pi_R^L(\beta_L) &= p^2\big[\pi(\beta_L, \beta_H) + \theta\beta_L\, q(\beta_H, \beta_L)\big] + p(1-p)\big[\pi(\beta_L, 0) + \beta_L\, q(0, \beta_L) + \pi(0, \beta_H)\big] \\ &\quad + (1-p)^2\pi(0,0) - \beta_L^2/2.\end{aligned}$$

Both are concave quadratic functions of $\beta_L$, maximized at

$$\begin{aligned}y_F^*(\beta_H) &= \frac{2p\big[(2-\gamma)(4-\gamma) - p(1-\theta)(2-\gamma)^2 + p\big(\theta(4-2\gamma+\gamma^2) - 2\gamma\big)\beta_H\big]}{(4-\gamma^2)^2 - 2p\,(8-8\gamma+\gamma^2) - 2p^2\big(4\theta^2 - 4\gamma\theta + \gamma^2\theta^2 + 8\gamma - \gamma^2 - 4\big)}, \\ y_R^*(\beta_H) &= \frac{p\big[(2-\gamma)(8-\gamma^2) - p(1-\theta)(2-\gamma)^2(2+\gamma) - 2p\big(2\gamma - \theta(4-\gamma^2)\big)\beta_H\big]}{(4-\gamma^2)^2 - 2p\,(4-4\gamma+\gamma^3) - 2p^2(1-\theta)\gamma(4-\gamma^2)}.\end{aligned}$$

The leader–follower equilibrium solves the linear system $\beta_H = x^*(\beta_L)$, $\beta_L = y^*(\beta_H)$. We verify that it satisfies $\beta_L < (1-\theta)\beta_H$ and that neither firm gains by deviating to the other branch of its best response. Close to $\theta_s$, the symmetric and the asymmetric configurations are not sharply separated. Just above $\theta_s$, an asymmetric equilibrium can coexist with the symmetric one, and we select the symmetric equilibrium; we verify that selecting the asymmetric equilibrium instead would raise the threshold, where it coincides with $\theta_s$, by less than 0.01 under fixed fees and 0.03 under royalties. Just below $\theta_s$, when products are differentiated, there may be no equilibrium in pure strategies; the direction of innovation is nonetheless determined, since we verify that, for any investment of the rival within the range of its best responses, each firm can secure along non-overlapping trajectories an expected profit above $\Pi^{o*}$, so that firms differentiate whichever equilibrium, possibly in mixed strategies, is played. Both cases arise only for values of $\theta$ very close to $\theta_s$.

**Direction and welfare.** Below $\theta_s$, we set $\Pi^{n*}$ equal to the average of the leader's and the follower's expected profits, and compute $W^{n*}$ at the profile $(\beta_H, \beta_L)$. In the intervals without pure-strategy equilibria, firms differentiate, as discussed above, so that these intervals do not belong to the failure region; we do not characterize welfare there. Under each contract, the threshold ($\theta_F^*$ or $\theta_R^*$) is the lowest $\theta$ such that $\Pi^{n*}(\theta') \geq \Pi^{o*}$ for every $\theta' \geq \theta$; it may thus lie below $\theta_s$.

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# Appendix C. Bargaining in cross-licensing

Online appendix to "Technology Licensing and the Direction of Innovation" by Fabio M. Manenti and Luca Sandrini.

In this appendix, instead of the take-it-or-leave-it offers assumed in the paper, the terms of each cross-license are set by bilateral Nash bargaining with equal weights, taking the other license as given (Nash-in-Nash bargaining). One-way licensing is as in the paper. The aim is to show that the results of the paper do not depend, qualitatively, on how bargaining power is distributed in cross-licensing. Above the replacement bound $\theta_s$ (Section 5 of the paper), the results are essentially unchanged. Below it, equal bargaining power works in favor of differentiation, so that the region in which the directional inefficiency survives is, overall, smaller than in the paper.

Section C.1 describes the bargaining protocol. Section C.2 reports the results in the Cournot model, with the same parameter values as in the paper ($a - c = 1$, $p = 1/2$, $I(\beta) = \beta^2/2$).

## C.1 The bargaining protocol

As in the paper, there are two licenses, one for each direction of the technology exchange, and the effective qualities under cross-licensing are $\hat{\beta}_i = \max\{\beta_i + \theta\beta_j, \beta_j\}$ and $\hat{\beta}_j = \max\{\beta_j + \theta\beta_i, \beta_i\}$, as in (1). Each license is negotiated bilaterally, taking the other license as given. In the negotiation over firm $i$'s technology, the disagreement outcome is that firm $i$ does not grant its technology, while the license on firm $j$'s technology remains in place. These are the same counterfactuals used in (3) and (4) to construct the licensor's minimum compensation and the licensee's maximum willingness to pay. The terms of each license maximize the product of the two parties' gains with respect to disagreement. This solution concept, in which each bilateral negotiation is a Nash bargaining problem given the outcome of the other negotiations, is commonly referred to as Nash-in-Nash bargaining (Horn and Wolinsky, 1988; Collard-Wexler et al., 2019). The complete agreement is implemented only if each firm prefers it to no licensing; otherwise neither license is signed and firms earn $\pi_i(\beta_i, \beta_j)$ and $\pi_j(\beta_j, \beta_i)$.

**Fixed fees.** With lump-sum payments, utility is transferable and the Nash product is maximized at the fee that splits the gains equally. Denoting by $\underline{L}_i^x$ the licensor's minimum compensation in (3) and by $\overline{L}_i^x$ the licensee's maximum willingness to pay in (4), the fee for the license on firm $i$'s technology is

$$L_i^x = \frac{\underline{L}_i^x + \overline{L}_i^x}{2}. \tag{C.1}$$

In the Cournot model, since $\pi(x, y) - \pi(y, x) = (x-y)(2+x+y)/(4-\gamma^2)$, the net transfer depends only on how much each firm's quality improves through the rival's technology:

$$L_i^x - L_j^x = \frac{(\hat{\beta}_j - \beta_j)(2 + \hat{\beta}_j + \beta_j) - (\hat{\beta}_i - \beta_i)(2 + \hat{\beta}_i + \beta_i)}{2(4 - \gamma^2)}, \tag{C.2}$$

which reduces to $\theta(\beta_i - \beta_j)[2 + \theta(\beta_i + \beta_j)]/[2(4 - \gamma^2)]$ when no replacement takes place. This is also the transfer the firms would obtain by negotiating a single agreement over reciprocal access, with equal weights and no licensing as the disagreement outcome. The

net transfer is zero at symmetric qualities, but it increases with a firm's own quality: at $\beta_i = \beta_j = \beta$, its derivative with respect to $\beta_i$ is $\theta(1+\theta\beta)/(4-\gamma^2)$.

**Royalties.** In the Cournot model, a per-unit royalty acts as a reduction in the effective quality of the firm that pays it: a firm operating an effective quality $x$ that pays $r$ per unit has margin $a-c+x-r$. With $q(x,y)$ and $\pi(x,y) = q(x,y)^2$ as in (13), where $x$ and $y$ are effective qualities net of royalties, each firm's payoff is its operating profit plus the royalty it collects on the rival's output.

Consider the license on firm $i$'s technology at rate $r$, given that the license on firm $j$'s technology is in place at rate $r_j^x$. Firm $i$ operates $x_i = \hat{\beta}_i - r_j^x$ whether or not it grants its own technology, while firm $j$ operates $x_j(r) = \hat{\beta}_j - r$ if it takes the license and $\beta_j$ otherwise. The gains with respect to disagreement are the royalty counterparts of (3) and (4). For the licensor,

$$G_i(r) = \underbrace{\pi\big(x_i, x_j(r)\big) - \pi(x_i, \beta_j)}_{\text{loss from a stronger rival}} + \underbrace{r\, q\big(x_j(r), x_i\big)}_{\text{royalty income}}, \tag{C.2}$$

and for the licensee,

$$G_j(r) = \underbrace{\pi\big(x_j(r), x_i\big) - \pi(\beta_j, x_i)}_{\substack{\text{gain from the absorbed technology,} \\ \text{net of the royalty}}} - \underbrace{r_j^x \big[q(x_i, \beta_j) - q\big(x_i, x_j(r)\big)\big]}_{\substack{\text{lower income} \\ \text{on its own license}}}. \tag{C.3}$$

The rate affects outputs only through $\hat{\beta}_j - \beta_j - r$, the improvement in the licensee's net quality. The licensee's gain is therefore zero at $r = \hat{\beta}_j - \beta_j$, where the royalty absorbs the whole value of the technology, and positive for lower rates on the non-drastic domain. As in Section 6.2, the highest rate the licensee accepts is $\overline{r}_i^x = \hat{\beta}_j - \beta_j$, which equals $\theta\beta_i$ absent replacement and $\beta_i - \beta_j$ when firm $j$ replaces its own technology with $i$'s. The rate on firm $i$'s technology is

$$r_i^x = \arg\max_{r \in [0, \overline{r}_i^x]} G_i(r)\, G_j(r), \tag{C.4}$$

and symmetrically for firm $j$. Because the licensee's gain vanishes at $\overline{r}_i^x$, the solution satisfies $r_i^x < \overline{r}_i^x$ whenever $\overline{r}_i^x > 0$.

Both gains are quadratic in $r$, so that the first-order condition of (C.4) is a cubic equation in $r_i^x$, whose coefficients depend on the reverse rate $r_j^x$. Its explicit solution is too cumbersome to be informative, and the first-order condition for investment, which depends on how both rates respond to a firm's own quality, has no tractable closed form. Unlike in the paper, where $r_i^x = \overline{r}_i^x$ and, for $\theta \geq \theta_s$, investment has the closed form (18), the rates and the non-overlapping investment are therefore computed numerically. In our computations the pair of rates is unique: iterating from different starting values gives the same solution.

Two remarks are in order. First, unlike lump-sum fees, royalties change the joint surplus of the two firms, because they raise marginal costs and soften competition. Bilateral negotiation over each license rules out the joint choice of rates studied by Jeon and Lefouili (2018): each rate must leave the licensee at least as well off as without that license, so it cannot exceed the value of the technology transferred. Second, the protocol differs from the one in the paper only in the cross-licensing event. One-way royalties remain equal to $r_i = \beta_i$, the highest rate the licensee accepts.

## C.2 Results

Expected profits are still given by (5)–(6) and (10)–(11), with the cross-licensing terms in (C.1) and (C.4). All the assumptions of Sections 2–4 hold in the Cournot model, as do Lemma 3, the regularity conditions of Section 3.3 and the participation conditions of both licenses, and equilibrium investments are non-drastic. As in the paper, for $\theta \geq \theta_s$ the investment stage has a symmetric equilibrium, and no unilateral deviation is profitable, once the complete continuation payoffs, including replacement, are taken into account; for $\theta < \theta_s$ it has leader–follower equilibria. We have verified these conditions numerically over a grid of values of $\gamma$ and $\theta$. The only assumption that changes is 5$c$) under royalties, which now holds strictly: each licensee pays less than the value it absorbs and expands output, so that consumers share in the gains from complementarity.

**Investment.** Investment along overlapping trajectories does not involve cross-licensing and is given by (14) under fixed fees and by (17) under royalties. Under fixed fees, investment along non-overlapping trajectories has, for $\theta \geq \theta_s$, the closed form

$$\beta^n = \frac{p(2-\gamma)\left[2(4-\gamma) - p(2-\gamma)(2-\theta)\right]}{(4-\gamma^2)^2 - 2p(8-8\gamma+\gamma^2) + 2p^2(4-6\gamma+\gamma^2) - p^2\theta(\theta+2)(2-\gamma)^2}, \tag{C.5}$$

which increases in $\theta$; under royalties it is computed numerically. As in Remarks 1 and 3, $\beta^n > \beta^o$ for every $\gamma$ and $\theta \geq \theta_s$ under both contracts.

**Replacement.** Proposition 4 holds unchanged: with perfectly substitute innovations there is no symmetric investment equilibrium under either contract. Under fixed fees, the marginal return to investment jumps upwards at the rival's investment level by $p^2\, d\pi(\beta,\beta)/d\beta$, exactly as in the paper: the equal split of the gain from replacement varies smoothly with the two qualities, and the jump comes only from the fact that the better technology is used by both firms. Under royalties the jump is slightly smaller than in the paper, but still positive. The bound $\theta_s$, however, is lower: at $p = 1/2$ it lies between 0.07 and 0.13 under both contracts, against 0.11 and 0.19 in the paper (Figure C.1). With equal weights, a firm that triggers replacement collects only half of the rival's gain from adopting its technology. Outspending the rival therefore pays less, and less complementarity is needed to sustain the symmetric equilibrium.

For $\theta < \theta_s$, the investment stage has leader–follower equilibria, as in the paper, but both firms invest less: under fixed fees, at $\gamma = 0.5$ and $\theta = 0$, the leader invests 0.475 and the follower 0.324, against 0.555 and 0.387 with take-it-or-leave-it offers. Since the leader's prize is halved, the contest between the two firms is milder, and the leader–follower configuration works as a division of labor: the follower saves on R&D, knowing that it will use the leader's technology if both firms succeed.[1]

**The direction of innovation.** Figure C.1 compares the thresholds with those of the paper. Above $\theta_s$, the threshold structure of Propositions 2 and 3 applies under both protocols, and the thresholds are close to those of the paper, slightly higher when products

[1] With equal weights, leader–follower equilibria also exist for $\theta$ above $\theta_s$, up to values of $\theta$ between about 0.15 and 0.25, where they coexist with the symmetric equilibrium. As in the paper, we select the symmetric equilibrium whenever it exists. Since the leader–follower equilibrium yields a higher average profit, selecting it instead could only widen the region in which firms differentiate, whereas with take-it-or-leave-it offers it would slightly raise the threshold (Appendix B of the paper).

are close substitutes.[2] More generally, for $\theta$ above $\theta_s$ the profit and welfare differentials have the same shape and sign as in Figures 1–4 of the paper, and the profit differentials under the two protocols nearly overlap. Below $\theta_s$, instead, equal bargaining power makes differentiation more attractive, since the milder contest between leader and follower dissipates less profit. Firms then differentiate for every $\theta$ over a wider range of product substitutability: up to $\gamma \approx 0.67$ under fixed fees and $\gamma \approx 0.58$ under royalties, against $\gamma \approx 0.43$ and $\gamma \approx 0.44$ in the paper. Above these values, the threshold is the one of Propositions 2 and 3, which increases in $\gamma$ up to 0.26 under fixed fees and 0.29 under royalties (0.22 and 0.26 in the paper).

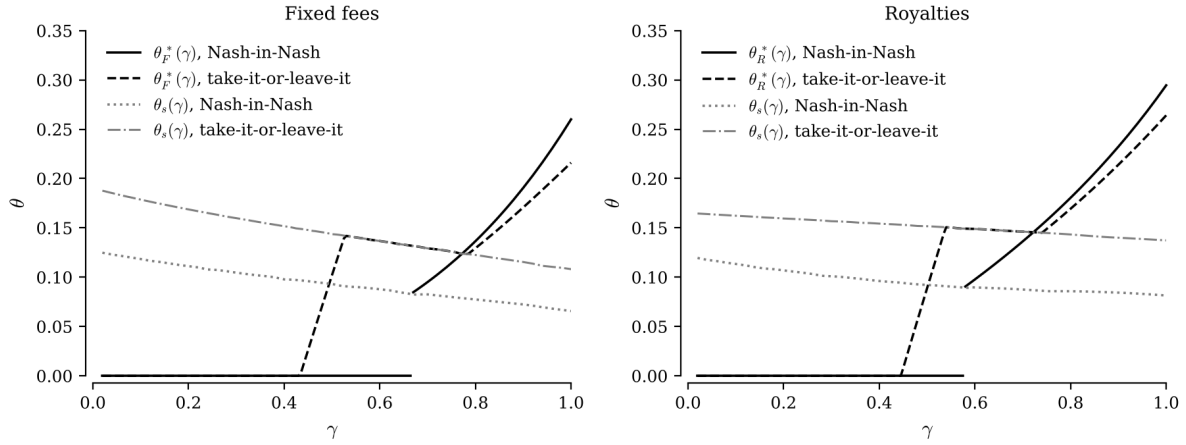



Figure C.1: Thresholds (black) and replacement bounds $\theta_s(\gamma)$ (grey) with Nash-in-Nash bargaining in cross-licensing and with take-it-or-leave-it offers as in the paper, $p = 1/2$. Left: fixed fees, $\theta_F^*(\gamma)$ (Figure 1, left panel). Right: royalties, $\theta_R^*(\gamma)$ (Figure 3, left panel).

For intermediate degrees of product substitutability ($0.67 \lesssim \gamma \lesssim 0.84$ under fixed fees, $0.58 \lesssim \gamma \lesssim 0.78$ under royalties), firms differentiate in the leader–follower region below $\theta_s$, converge just above $\theta_s$, where the symmetric equilibrium is restored while complementarity is still low, and differentiate again above the threshold. As in the paper (Appendix B), the threshold is the lowest $\theta$ above which firms always differentiate.

**Welfare and the failure region.** Expected welfare is higher under non-overlapping trajectories for every $\gamma$ and $\theta$ under both contracts, including the leader–follower region below $\theta_s$, and Corollaries 1 and 2 hold at the equilibrium investment levels. As in the paper, the market-failure region therefore coincides with the region where firms converge (Figure C.2): the directional inefficiency survives only when the innovations developed along the two trajectories are close substitutes, and it is quantitatively relevant only when products are close substitutes as well. With equal bargaining power, the failure region is smaller for intermediate degrees of product substitutability, where firms differentiate below $\theta_s$, and slightly larger for nearly homogeneous products. Overall, it is about half as large as in the paper under fixed fees, and about a third smaller under royalties.

[2]With equal weights, the net cross-licensing fee responds more strongly to a firm's own quality when products are close substitutes, which raises investment along non-overlapping trajectories and lowers equilibrium profits there. Under royalties, rates are lower and soften competition less. In both cases differentiation becomes slightly less profitable when products are close substitutes, while the value of overlapping trajectories is unchanged.

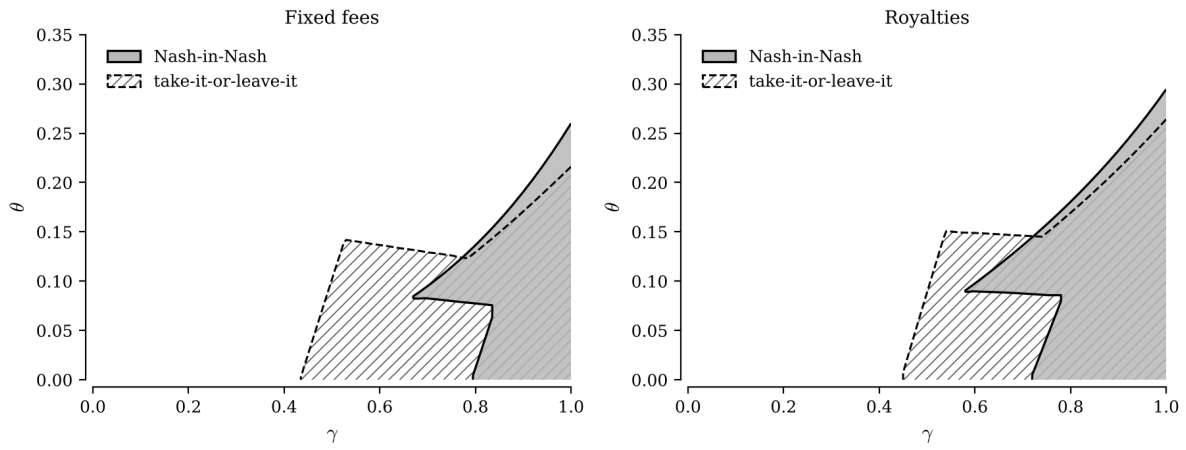



Figure C.2: The market-failure region, where firms choose overlapping trajectories while society prefers non-overlapping ones, $p = 1/2$, with Nash-in-Nash bargaining in cross-licensing (grey area, solid boundary) and as in the paper (hatched area, dashed boundary). Left: fixed fees (Figure 2, right panel). Right: royalties (Figure 4, right panel).

**Comparing the two contractual forms.** The three findings of Section 6.3 continue to hold. First, fixed fees align private and social incentives at least as well as royalties: with equal bargaining power, the royalty threshold is at least as high as the fixed-fee threshold for every $\gamma$, and strictly higher for $\gamma \gtrsim 0.58$ (Figure C.3). Second, fixed fees dominate royalties in terms of welfare for every $\gamma$ and $\theta$, while firms earn higher profits under royalties when products are sufficiently close substitutes. Third, for low complementarity there is a range of product substitutability in which firms differentiate under fixed fees and converge under royalties, so that the contract form determines the direction of innovation: with $\theta = 0.2$, this range is $\gamma \in (0.84, 0.92)$, against $(0.87, 0.96)$ in the paper.[3]

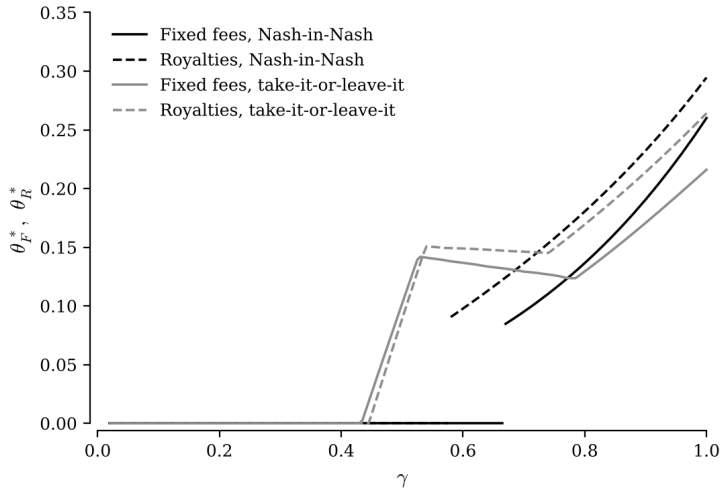



Figure C.3: Thresholds under fixed fees (solid) and royalties (dashed), $p = 1/2$, with Nash-in-Nash bargaining in cross-licensing (black) and as in the paper (grey; Figure 5, top left panel).

[3]We have repeated the analysis for $p = 0.3$ and $p = 0.7$: the results above hold qualitatively in both cases.

**Summary.** With Nash-in-Nash bargaining in cross-licensing, the Cournot model behaves as the model of the paper above the replacement bound $\theta_s$, and Proposition 4 holds unchanged. Below $\theta_s$, equal bargaining power halves the leader's gain from replacement and tempers the contest between leader and follower, so that firms differentiate over a wider range of product substitutability. In this respect, the take-it-or-leave-it offers of the paper are a conservative assumption: with equal bargaining power the failure region is smaller, except for nearly homogeneous products, where it is slightly larger.

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