Allocation, Costs and Prices in the Electricity Wholesale Market and Balancing Power Market – An Integrated Approach

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Abstract

This paper investigates the economic interdependencies between the electricity wholesale market and the balancing power market. We present an integrated market model that relates to the future harmonization of the European balancing power markets. We prove that there exists an overall market equilibrium, which ensures an efficient outcome under certain conditions, and we analyze the equilibrium allocation, prices and costs. We also show that lower prequalification criteria for the balancing power market reduce the total costs of the power system. By analyzing the market data from Germany we find that the costs decreased over the years, but they are still above the theoretical equilibrium.

Keywords: Balancing Power; Market Design; Market Equilibrium; Procurement Auction; Reserve Power

JEL: D2, D4, D5, D6, L1, L5

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1. Introduction

The continuous growth of renewable energy sources reduces the predicability of electric energy production substantially. To ensure a stable grid frequency (e.g. 50 Hertz in the synchronous grid of Continental Europe), ancillary services become increasingly important. The most relevant short-term ancillary service is balancing power (*BP*). In liberalized electricity markets, the procurement of BP is usually carried out by the Transmission System Operators (*TSOs*) or the regulatory authority. The applied market mechanism is mostly a procurement auction, in which prequalified suppliers compete for the provision of BP.

In our paper, we present an integrated market model to analyze the interdependencies between the wholesale electricity market and the BP market, which have been so far only partially considered in literature. By providing a better understanding of the interdependencies, our analysis may also help to appropriately coordinate future market designs.

There are empirical and theoretical analyses of BP markets. Rammerstorfer and Wagner (2009) empirically assess the effects of a reorganization of the German BP market. Heim and Götz (2013) present evidence for strategic capacity withholding by a supplier with market power. Hirth and Ziegenhagen (2015) discuss the implications of the connection of renewable energy sources to BP market. Ocker et al. (2016) and Ocker (2017) show that there is there is a heterogeneity among the market designs in Europe. Ocker and Ehrhart (2017) provide empirical evidence for collusive behavior in the German BP markets. Bushnell and Oren (1995) and Chao and Wilson (2002) theoretically analyze different scoring auctions and derive rules to ensure efficiency. Kamat and Oren (2002) analyze efficiency properties of alternative auction formats for BP markets in the USA. Müller and Rammerstorfer (2008) categorize several design elements for BP auctions. Wen and David (2002) derive optimal bidding strategies in a stochastic optimization model. Swider and Weber (2007) present a methodology for profit maximizing bidding under price uncertainty for power systems reserve. Müsgens et al. (2014) discuss the economic fundamentals that govern market design and behavior in the German BP markets. Ocker et al. (2018b) focus on the bidding strategies in the Austrian and German BP auctions. Ortner (2017) discusses fundamental approaches and illustrates case studies assuming perfect competition. Ocker et al. (2018a) present a game-theoretical analysis of the BP auctions and point out that the regular repetition of the auctions invite the suppliers to implicitly collude.

In most theoretical analyses, the interdependencies of the electricity wholesale market and the BP market are only partially considered. For example, Müsgens et al. (2014), Hirth and Ziegenhagen (2015) and Ocker et al. (2018a) assume that the wholesale market price is exogenous. This yields two classes of suppliers: suppliers with power plants that have variable cost below the wholesale market price, and suppliers with variable cost above the wholesale market price. The former sell electric energy profitably on the wholesale market, while the latter do not participate in this market. Müsgens et al. (2014) denote these two types of suppliers "inframarginal" and "extramarginal." This distinction has a direct impact on the costs for providing BP: inframarginal suppliers must integrate opportunity costs of not trading at the wholesale market, and extramarginal suppliers must cover their expenses by profits of the BP market (e.g. Hirth and Ziegenhagen, 2015; Ocker et al., 2018b).

To our knowledge, only Just and Weber (2008) address these interdependencies: suppliers of BP cannot offer their entire capacity on the wholesale market, however, have to run their plants at a certain minimal load. Applying a numerical solution procedure, the authors focus on the identification of reservation pricing and the impact of reserve capacity on the supply function of the wholesale market.

Our approach relates to the work of Just and Weber (2008), however, we use a different methodology for analyzing the interdependencies between the wholesale electricity market and the BP market. The interplay between the markets induces a specific assignment of the energy producers to the different markets according to the producers' production costs and their ability to provide BP. Thus, inframarginality and extramarginality are endogenously determined. There exists a unique market equilibrium that ensures efficiency under certain assumptions. We also consider prices and costs in the markets as well as the distribution of surpluses. The comparison with German market data indicates that the actual BP costs are higher than predicted by our model, which particularly applies to the costs of BP activation.

The remainder of this paper is structured as follows. Section 2 provides a brief overview of the electricity wholesale and BP market design. Section 3 presents our integrated market model. In Section 4 we analyze and discuss the overall market equilibrium and the effects of the BP prequalification conditions and market power. Section 5 contrasts the theoretical findings with German market data. Section 6 concludes and points at further need of research.

2. Electricity Markets

In this section we provide a general overview of the electricity markets.

2.1. Electricity Wholesale Market

Electric energy (henceforth energy) is traded at forward markets and at spot markets in most European countries (e.g. Ströbele et al., 2013; KU Leuven Energy Institute, 2015; Zweifel et al., 2017). Forward markets enable the trade of energy more long-term, whereas at spot markets the point of delivery is instantaneous, i.e., typically within the next 48 hours. Therefore, forward markets are mostly utilized for risk hedging, and trading is often carried out bilaterally (so-called "over the counter"). When trading at spot markets, there are two types of markets available: a "Day-Ahead" auction and an "Intraday Continuous" auction.[1] In the Day-Ahead auction, trading is done for the following day, whereas in the Intraday Continuous auction, trading is done only minutes before the actual delivery. The market clearing price is commonly uniform and determined by the last (i.e., highest) accepted bid.

[1] For an overview of the design of European spot markets see Ocker et al. (2016).

2.2. Balancing Power Market

The volumes traded at the wholesale market can differ substantially from the actual production of energy because they are based on predictions of supply and demand. A deviation directly influences the grid frequency in alternating current systems: if too much (little) energy is supplied, the grid frequency increases (decreases), which can lead to area-wide black-outs. For securing a reliable power system, most TSOs apply ancillary services for stabilizing the power grid. The most-important short-term ancillary service is BP (Müsgens et al., 2014; Hirth and Ziegenhagen, 2015; Zweifel et al., 2017).

There are three different qualities of BP: Primary BP (PBP), Secondary BP (SBP), and Tertiary BP (TBP) (e.g. entso-e, 2016; Ocker et al., 2016).[2] These are distinguished by the reaction time of BP being available: First, PBP is activated to limit deviations from the grid frequency, then SBP is utilized to restore the grid frequency and, finally, TBP is activated as a more long-term measure. The reaction times differ across Europe (e.g. Ocker et al., 2016). In Germany, for example, the reaction times are as follows: PBP must be available after 30 seconds until 5 minutes, SBP after 5 minutes until 15 minutes, and TBP after 15 minutes until 60 minutes after an imbalance. The three qualities have separate markets that are organized as procurement auctions (Ocker et al., 2016). We focus on the SBP market because it has the highest demand and is the most important short-term ancillary service (Borne et al., 2018). Since the SBP market is to be harmonized across Europe no later than 2021, we refer to the future European auction design in our analysis (European Commission, 2017).

A deviation of the predicted production schedule can either lead to an overproduction (e.g. from wind plants during a storm) or an underproduction (e.g. from solar plants during a cloudy day). Consequently, BP needs to provide both an increased and decreased energy

[2]PBP is also known as Frequency Containment Reserve (FCR), SBP as automatically-activated Frequency Restoration Reserve (aFRR), and TBP as manually-activated Frequency Restoration Reserve (mFRR).

supply. This is achieved by implementing two different BP products: in the positive market, suppliers provide upward regulation, while in the negative market, suppliers provide downward regulation (e.g. entso-e, 2016; Ocker et al., 2016).

Since suppliers of BP need to adjust the load level of their power plants within seconds, the provision of BP requires a high degree of operational and technical flexibility. Hence, an elaborate prequalification process for the BP market participation is required. As a consequence, the supply set is highly invariant and limited.[3]

Providing BP comes with costs for the suppliers: they have to be compensated for keeping BP (Megawatt, MW) available to the grid and also for the actual delivery of balancing energy (BE) (Megawatt hour, MWh). Suppliers submit three-dimensional bids in the SBP auction: a power offer (with unit MW), a power bid (with unit Euro/MW), and an energy bid (with unit Euro/MWh). Since TSOs are legally forced to procure BP at the lowest possible costs, they calculate scores for the suppliers and those with the lowest scores are awarded. In the future harmonized SBP market, TSOs may base these scores on both the power and energy bid, only the power bid or only the energy bid (European Commission, 2017).[4]

The awarded suppliers' power and energy price are determined via pricing rules. Two pricing rules are applied: Pay-as-bid (PaB) or uniform pricing (UP). If PaB is used, awarded suppliers are paid their submitted bids. If UP is used, all awarded suppliers are paid a uniform price. In the future harmonized SBP market, UP is to be implemented (entso-e, 2016).

For the determination of the uniform price (market clearing price), two rules are usually applied in practice: the price is determined either by the last (highest) accepted bid or by the first (lowest) rejected bid (e.g. Kahn et al., 2001; Müsgens et al., 2014; Ocker et al.,

[3]For the German markets, see for example regelleistung.net (2017) for the prequalification criteria and Kaut et al. (2017) for an analysis of market concentration.

[4]In the theoretical analysis we relate to the scoring rule that considers only the power bid, since it is commonly applied across Europe (Ocker et al., 2016).

2018a). The fact that the first rule is more prevalent in practice should be interpreted as a convention. If a bidder's probability to determine the price with one of her bids is small, e.g. because the number of bidders is large, the strategic incentives are almost equal under both rules. Moreover, in an efficient auction equilibrium, both rules (and other pricing rules) lead to the same expected outcome, which includes the same allocation, the same expected bidders' profits, and the same expected auction revenue/costs (Engelbrecht-Wiggans, 1988).

The delivery of BE is activated according to a merit-order of the energy bids, which discriminates with respect to the activation duration of the power plants (European Commission, 2017). That is, the suppliers with the lowest energy bids are utilized first.

3. Integrated Market Model

3.1. Interdependencies

BP is provided by prequalified suppliers that have to meet specific technical requirements, i.e., a certain degree of technical flexibility regarding the operation mode of their power plant.[5] Since not all types of power plants are qualifiable, there is a coexistence of two distinct types of suppliers on the wholesale market: suppliers that exclusively offer their capacities in the wholesale market, and suppliers which can offer their capacities on both the wholesale market and the BP market. Thus, there are fundamental interconnections between the two markets because BP suppliers cannot offer their entire capacity on the wholesale market, however, must run their power plants at a certain minimal load. This results in must-run capacities, whose energy is sold on the wholesale market (Just and Weber, 2008).

3.2. The Model

There are three energy markets: a wholesale market, a positive and a negative BP market. We consider a certain period (e.g. year). The average demand on the wholesale market in

[5] Depends on the criteria determined by the country, BP quality, etc. For Germany, see the official website of the four German TSOs (regelleistung.net, 2017).

this period is denoted by D and measured in Gigawatt (GW). The (capacity) demand on the positive and negative BP market is fixed and given by B^+ and B^- (with $B = B^+ + B^-$).

There is a set of energy suppliers. Each supplier participates in the markets with one power plant. The plants have the same capacity but differ in their variable energy production costs, which lie in the interval $[\underline{c}, \overline{c}]$. We assume that UP is implemented on all considered markets, which is the common pricing rule on the wholesale market and considers the intentions for a harmonized European SBP market (European Commission, 2017). That is, all awarded suppliers receive the marginal price. On the BP markets, the uniform price for BE is repeatedly determined within a pre-defined period of time – the "Balancing Energy Pricing Period" (*BEPP*). The application of UP allows to assume that the suppliers reveal their cost in their bids.[6] Thus, the supply function $S : [\underline{c}, \overline{c}] \rightarrow \mathbb{R}^+$ is strictly increasing, where $S(c)$ is the supply at price c. The inverse function is $S^{-1} : \mathbb{R}^+ \rightarrow [\underline{c}, \overline{c}]$. There are two types of suppliers: BP-capable (*BP*) suppliers and non-BP-capable (*nBP*) suppliers. The nBP suppliers only participate on the wholesale market, while BP suppliers can participate on the wholesale market and the BP market. For the latter, they must run their plant on a minimal load (i.e., share of capacity) $m \in [0, 1)$ and sell this energy on the wholesale market. The supply includes BP and nBP suppliers: $S(c) = S_{BP}(c) + S_{nBP}(c)$, where $S_{BP} : [\underline{c}, \overline{c}] \rightarrow \mathbb{R}^+$ and $S_{nBP} : [\underline{c}, \overline{c}] \rightarrow \mathbb{R}^+$ denote the supply functions of BP and nBP suppliers. We assume

[6] For the wholesale market, this is the usual and acceptable assumption. Things are different on the BP markets because here the time a supplier delivers BE depends on her merit-order position. As a consequence, the length of the BEPP has an impact on the suppliers' bidding strategy. The longer the BEPP, the more bids are taken into account for the determination of the price, which induces the suppliers to reduce their energy bids, even below their costs (Ocker et al., 2018a,b). Under the realistic condition that the last accepted bid determines the price under UP, a very short BEPP causes a high probability for a bidder that her awarded bid determines the price. In this case, which converges towards the PaB case, the suppliers have an incentive to exaggerate their costs in their bids. Thus, there exists a BEPP which incentivizes the suppliers to bid their costs. Nevertheless, applying the "Revenue Equivalence Theorem" (e.g. Krishna, 2002), the expected equilibrium outcome (i.e., allocation, suppliers' expected profits, expected (average) prices, and expected BP costs) neither depends on the pricing rule (PaB or UP) nor on the length of the BEPP under UP (see Ocker et al. (2018a) and Section 2.2). For these reasons and the sake of clarity and simplicity, we take the liberty to assume truthful bidding, i.e., the suppliers bid their true costs.

that BP suppliers are uniformly distributed among all suppliers: at each cost level c, the BP suppliers' share of the supply $S(c)$ is $\delta \in [0,1]$ and, thus, the nBP suppliers' share is $1-\delta$.

Discrepancies between demand and supply are balanced by calling BP. The functions

$$\begin{aligned} z^- &: (0, B^-] \to [0,1] \\ z^+ &: [0, B^+] \to [0,1] \end{aligned} \tag{1}$$

describe the cumulated relative frequency of the differences between demand and supply in the period: $z^-(x)$ is the relative frequency of an excess supply of at least x GW. Thus, $z^-(x)$ refers the call of negative BP of at least x GW. Analogously, $z^+(x)$ applies to an excess demand and, thus, to the call of positive BP.[7]

Thus, the z-functions (1) also describe the relative calling frequencies (abbrev.: rcf), i.e., the relative frequencies of calling BP: $z^-(x)$ or $z^+(y)$ is the share of time within the period where a minimum capacity of x negative BP is called or y positive BP is called, respectively. The rcf of 2015 are shown in Figure 1. The interval [-2,000, 0] MW belongs to the negative SBP market, i.e., $B^- = 2$ GW (note that the curve of z^- is shown reversely in the figure), while the interval [0, 2,000] MW belongs to the positive SBP market, i.e., $B^+ = 2$ GW.

The z-functions (1) are strictly decreasing with $z^-(B^-) = z^+(B^+) = 0$ and $z^-(x) + z^+(y) \leq 1$ for $x \in (0, B^-]$ and $y \in [0, B^+]$. The integrals

$$\tilde{B}^- = \int_0^{-B^-} z^-(x)dx \ \text{ and } \ \tilde{B}^+ = \int_0^{B^+} z^+(x)dx \tag{2}$$

are the expected total negative BP capacity and the expected total positive BP capacity that are needed to balance the excess supply and excess demand. Thus, $\gamma^- = \tilde{B}^-/B^-$ and

[7] We assume that the discrepancies are only caused by supply fluctuations due to production deviations, which can be justified by the increasing penetration of variable renewable energy sources into the power system. Their energy production depends on the weather conditions and is therefore highly volatile.

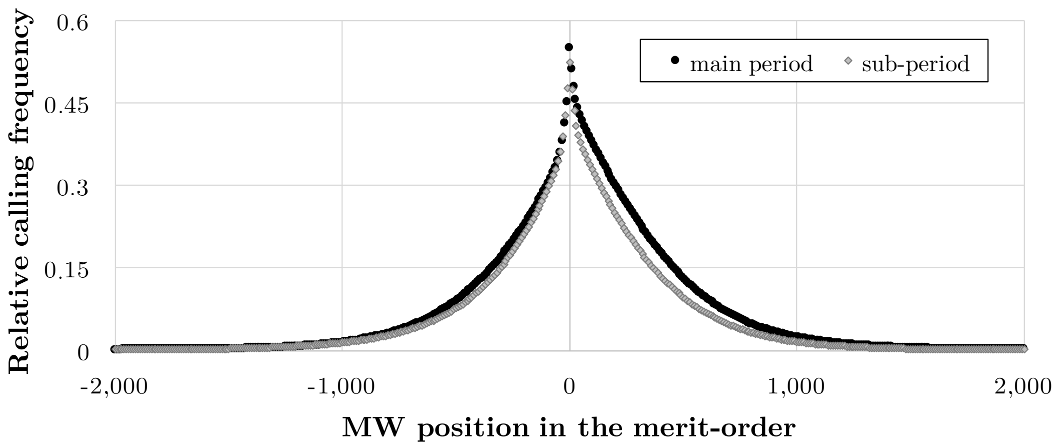

Figure 1: Empirical functions for the rcf in the German positive (interval [0, 2,000] MW) and negative (interval [-2,000, 0] MW) SBP market (main period and sub-period) (50hertz, 2017).

$\gamma^+ = \tilde{B}^+/B^+$ are the fraction of provided negative BP and of positive BP, i.e., the fraction of the BP capacities for the delivery of BE demand (with $\tilde{B} = \tilde{B}^+ + \tilde{B}^-$). We call the BP markets symmetric if the difference between demand and supply on the wholesale market is symmetrically distributed, i.e., $B^- = B^+ = \frac{B}{2}$ and $z^-(x) = z^+(y)$ for $x = y$. In this case, $\tilde{B}^- = \tilde{B}^+$ and the average demand and average supply are equal.

Let c_0^+ (c_0^-) denote the lowest variable cost of all suppliers on the positive (negative) BP market and c_1^+ (c_1^-) the highest variable cost.[8] The BP merit-order maps a supplier (according to her cost c) onto a merit-order position on the positive market by the bijective function $r^+ : [c_0^+, c_1^+] \to [0, B^+] =: R^+$ and on the negative market by $r^- : [c_0^-, c_1^-] \to [B^-, 0] =: R^-$. Each rank is assigned a rcf by the mappings $a^+ : R^+ \to [a^+_{max}, a^+_{min}]$ and $a^- : R^- \to [a^-_{min}, a^-_{max}]$. The rcf determines the average share of time in which the supplier delivers BE. The values a^+_{max} and a^-_{max} (a^+_{min} and a^-_{min}) denote the highest (lowest) rcf in the

[8]Any supply fluctuation demands the provision of BP. The respective costs are accounted to the suppliers. In our theoretical model, we do not account for these additional costs since they reflect on average approximately 0.1% of the suppliers variable cost (see Section 5.2). We assume that any supply fluctuation triggers SBP, and that all suppliers of the wholesale market deviate identically. For this, dc denotes the average cost of BE per MW caused by supply fluctuations. The sum $c + dc$ represents the imputed variable cost.

two BP markets. The rcf are determined by the z-functions (1), where

$$\begin{aligned}
a^+_{min} &= z^+(B^+) = 0,\\
a^+_{max} &= z^+(0) \in (0,1),\\
a^-_{min} &= z^-(B^-) = 0,\\
a^-_{max} &= \lim_{x\downarrow 0} z^-(x) \in (0,1),\\
a^+_{max} &+ a^-_{max} \leq 1.
\end{aligned} \tag{3}$$

On the wholesale market, suppliers constantly produce energy and are remunerated for each unit by the wholesale market price p_S. On the BP markets, a supplier receives the BP price p^+_{BP} or p^-_{BP}, and, if she is called, additionally a BE price $p^+_{BE}(c)$ or $p^-_{BE}(c)$. The BP prices p^+_{BP} and p^-_{BP} are the same for all suppliers and are determined by the highest accepted power bid in the positive respectively negative BP market. The BE price $p^+_{BE}(c)$ ($p^-_{BE}(c)$) is determined by the associated costs of the highest merit-order position that is needed to cover the demand within the BEPP. Thus, the length of the BEPP influences the supplier's BE prices: the longer the BEPP, the higher is the number of draws for BE demand, and, thus, the higher (lower) are the cost of the last supplier on the positive (negative) market. We model the average supplier's BE price in dependence of factor $\vartheta \in (0,1]$ that corresponds to the length of the BEPP:

$$p^+_{BE}(c) = c\,(1-\vartheta) + c^+_1\,\vartheta\,, \tag{4}$$

$$p^-_{BE}(c) = -c\,(1-\vartheta) - c^-_0\,\vartheta\,. \tag{5}$$

The case $\vartheta = 1$ models the longest possible BEPP, in which the BE price p^+_{BE} (p^-_{BE}) is always determined by the highest (lowest) supplier's cost c_1^+ (c_0^-).[9] The smaller ϑ (i.e., the shorter the BEPP), the closer moves the supplier's average BE price to her cost c.

A supplier's profit per produced energy unit on the wholesale market is[10]

$$\pi_S(c) = p_S - c\,, \tag{6}$$

on the positive BP market

$$\begin{aligned}\pi_{BP}^+(c) &= m(p_S - c) + (1-m)p_{BP}^+ + (1-m)a(r(c))(p_{BE}^+(c) - c)\\ &= m(p_S - c) + (1-m)[p_{BP}^+ + a(r(c))(p_{BE}^+(c) - c)]\,,\end{aligned} \tag{7}$$

and on the negative BP market

$$\begin{aligned}\pi_{BP}^-(c) &= m(p_S - c) + (1-m)p_{BP}^- + (1-m)[(p_S - c) + a(r(c))(c + p_{BE}^-(c))]\\ &= (p_S - c) + (1-m)[p_{BP}^- + a(r(c))(c + p_{BE}^-(c))]\,.\end{aligned} \tag{8}$$

Equation (6) states the difference of the wholesale market price and a supplier's variable cost. In the positive BP market, the profits consist of two parts. The first part of (7) represents the profits from selling the minimal load on the wholesale market, and the second part states the profits generated by the BP price and the BE price (depending on the rcf). In the negative BP market, suppliers are continuously paid the wholesale market price for their entire capacity. Recall that a supplier provides negative BP by decreasing the load level of her power plant, since there is an oversupply to the power system. Consequently, the provision of negative BP has no impact on her trading on the wholesale market. Therefore,

[9] Note that suppliers submit negative energy bids in the negative market (see (8)).
[10] The wholesale market price fluctuates and, thus, the supplier's profit, i.e., p_S and $\pi_S(c)$ are average values.

the first part of (8) represents the margin of selling their entire capacity at the wholesale market. The second part states the BP profits, which consist of the payment for BP and BE.

3.3. Conditions for Efficiency, Stability and Market Clearing

In this section we present some crucial conditions for our model.

3.3.1. Efficient Allocation on the BP Markets

An efficient allocation on the BP markets requires that plants with low variable cost are preferred to plants with high variable cost for the production of an additional unit of energy. Thus, plants with low variable cost must have higher production volumes than plants with high variable cost (Müsgens et al., 2014). This yields a unique order of c_0^+, c_0^-, c_1^+ and c_1^-.

In the negative BP market, c_0^- denotes the supplier on the last rank in the merit-order with a rcf of $a_{min}^- = 0$, i.e., she continuously produces with her entire capacity of size 1. The supplier with c_1^- is assigned the rcf of a_{max}^-, i.e., her plant operates with the load $m + (1 - a_{max}^-)(1 - m) < 1$. This yields $c_0^- < c_1^-$.

In the positive BP market, c_0^+ is assigned to the supplier on the first rank in the merit-order with rcf a_{max}^+. Thus, her plant operates on the load level $m + a_{max}^+(1-m) \leq m + (1 - a_{max}^-)(1 - m)$ because of $a_{max}^+ + a_{max}^- \leq 1$. This yields $c_1^- = c_0^+$. The supplier with c_1^+ is on the last rank of the merit-order. She never provides BP, but runs her plant permanently on the load level m on the wholesale market. As a result, we get the following order.

$$\textbf{(A0) Rank of costs} \quad c_0^- < c_1^- = c_0^+ < c_1^+$$

Note that this order in conjunction with (4) and (5) yield $|p_{BE}^-| < p_{BE}^+$.

3.3.2. Stability Criteria

BP suppliers either participate at the wholesale market or the BP market. This raises the question about the stability of the efficient BP allocation. That is, do prices exist such that the suppliers are incentivized to choose the "right" position of their own accord?

The conditions for the suppliers on the first and last position in the merit-orders are most crucial. The supplier with c_0^- has to be indifferent between her last position in the merit-order of the negative BP market and a switch to the wholesale market. The supplier with c_1^+ must be indifferent between her last position in the merit-order of the positive BP market and not participating at all. The suppliers with c_1^- (c_0^+) need to be indifferent between a switch to the positive (negative) BP market. This leads to the following stability conditions.

(M0) Between-market $\quad (p_S - c_0^-) + (1-m)p_{BP}^- \stackrel{!}{=} p_S - c_0^-$

(M1) Market-entrance $\quad m(p_S - c_1^+) + (1-m)p_{BP}^+ \stackrel{!}{=} 0$

(M2) BP-markets $\quad (p_S - c_1^-) + (1-m)(p_{BP}^- + a_{max}^-(c + p_{BE}^-(c)))$
$$\stackrel{!}{=} m(p_S - c_0^+) + (1-m)(p_{BP}^+ + a_{max}^+(p_{BE}^+(c) - c_0^+))$$

If one of these conditions is violated, either producing suppliers have an incentive to switch markets or non-producing suppliers have an incentive to enter the BP market.

3.3.3. Market Clearing and Energy Balance

Since BP suppliers only use the share $1-m$ of their capacities to provide BP, their total capacity to cover B has to be $\frac{B}{1-m}$. According to the considerations in 3.2 and 3.3.1, this capacity is provided by the BP suppliers with costs between c_0^- and c_1^+. In order to cover B^-, a total capacity of $\frac{B^-}{1-m}$ is needed, which is provided by the BP suppliers with costs between c_0^- and c_1^-. The BP suppliers in the subsequent cost interval $[c_0^+, c_1^+]$ with $c_0^+ = c_1^-$ together provide $\frac{B^+}{1-m}$ for covering B^+. Hence, the interval $[c_0^-, c_1^+]$ corresponds to cumulated BP capacities in the interval $[0, \frac{B}{1-m}]$, where $[c_0^-, c_1^-)$ corresponds to $[0, \frac{B^-}{1-m})$ and $[c_0^+, c_1^+]$ to $[\frac{B^-}{1-m}, \frac{B}{1-m}]$. Within the interval $[0, \frac{B^-}{1-m})$, the rcf increases from $a_{min}^- = 0$ to a_{max}^-. Thus, since we are on the negative BP market, the suppliers' (expected) active capacities for providing energy for the BP markets and the wholesale market decrease from 1 to $1-(1-m)a^-_{max}$. Within the interval $[\frac{B^-}{1-m}, \frac{B}{1-m}]$, which applies to the positive BP market, the rcf decreases from a^+_{max} to $a^+_{min}=0$, and thus, the suppliers' active capacities decrease from $m+(1-m)a^+_{max}$ to m. Due to the condition $a^-_{max}+a^+_{max}\leq 1$, $1-(1-m)a^-_{max}\geq m+(1-m)a^+_{max}$ and, thus, the curve of active capacities is strictly decreasing within $[0, \frac{B}{1-m}]$. Applying the z-functions (1) together with (3), the curve of the BP suppliers' active capacities for providing energy for the BP markets and the wholesale market is given by the strictly decreasing function $j(q)$:

$$j(q)=\begin{cases} m+(1-m)\left(1-z^-(B^--q(1-m))\right) & : q\in[0, \frac{B^-}{1-m}) \\ m+(1-m)\,z^+(q(1-m)-B^-) & : q\in[\frac{B^-}{1-m}, \frac{B}{1-m}]\,. \end{cases} \tag{9}$$

Using the maximum rcf a^+_{max} leads to

$$j(q)=\begin{cases} 1 & : q=0 \\ m+(1-m)\,a^+_{max} & : q=\frac{B^-}{(1-m)} \\ m & : q=\frac{B}{1-m}\,. \end{cases} \tag{10}$$

The integral

$$J(m,B)=\int_0^{\frac{B}{1-m}} j(q)dq \tag{11}$$

is the average active capacity of all BP suppliers. With (2) we get

$$J(m,B)=\frac{B^-}{1-m}-\tilde{B}^-+\frac{mB^+}{1-m}+\tilde{B}^+\,. \tag{12}$$

Figure 2 illustrates the symmetric case with $j(\frac{B}{2(1-m)})=\frac{1+m}{2}$, $j(\frac{B}{1-m}-q)=1+m-j(q)$

for $q \in [0, \frac{B}{2(1-m)}]$, and

$$J(m, B) = \frac{B\,(1+m)}{2(1-m)}\,. \tag{13}$$

That is, in the symmetric case, $J(m, B)$ is independent of the shape of the j-curve. Since the asymmetry in the German SBP markets is small (see Figure 1), we take the liberty to restrict the following analysis to the symmetric case.

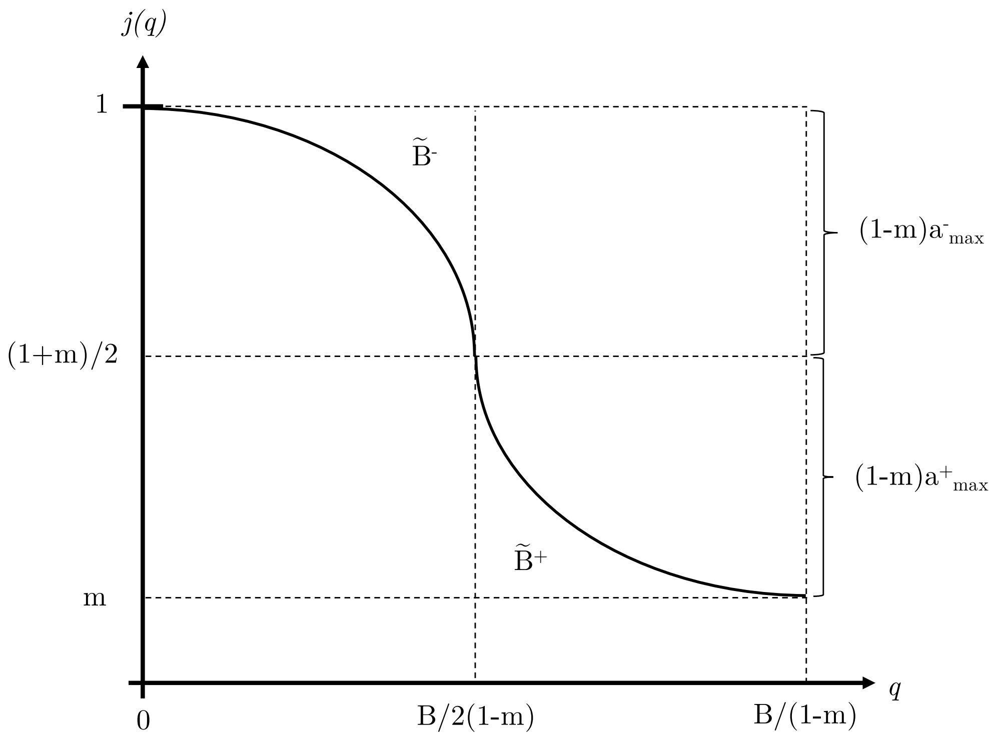

Figure 2: Example of a j-function in a symmetric BP market.

Market clearing and energy balance require the following conditions:

(S0) Wholesale market $D = S_{nBP}(p_S) + S_{BP}(c_1^-) + \frac{mB^+}{1-m}$

(S1) Positive BP market $B^+ = (1-m)(S_{BP}(c_1^+) - S_{BP}(c_0^+))$

(S2) Negative BP market $B^- = (1-m)(S_{BP}(c_1^-) - S_{BP}(c_0^-))$

(S3) Energy balance $D - S_{nBP}(p_S) + S_{BP}(c_0^-) + \tilde{B}^- - \tilde{B}^+ + J(m,B) = 0$

The demand D on the wholesale market is met by the contracted supply, which refers to the case without deviations (S0). This supply is provided by nBP suppliers and BP suppliers. The supply $S_{nBP}(p_S)$ includes all nBP plants with cost between $\underline{c}$ and p_S. The supply of the BP plants comprises of two groups: the contracted supply of the negative BP suppliers with cost between $\underline{c}$ and c_1^- is $S_{BP}(c_1^-)$ and that of the positive BP suppliers with cost between c_0^+ and c_1^+, whose contracts only refer to the minimal load m, is $\frac{mB^+}{1-m}$. The demand for positive and negative BP is provided by the $(1-m)$th share of BP plants within the interval $[c_0^+, c_1^+]$ and $[c_0^-, c_1^-]$ (S1, S2). Condition (S3) requires that total supply meets demand D also in case of deviations. Positive deviations $\tilde{B}^+$ and negative deviations $\tilde{B}^-$ are balanced by the BP suppliers with cost between c_0^- and c_1^+, whose active capacities are given by $J(m,B)$.

3.4. Total Costs of the Integrated Power System

The total costs C of the integrated power system are

$$C = \int_{(1-\delta)\beta}^{D-q_{BP}-q_0} S_{nBP}^{-1}(q)\,dq + \int_{\delta\beta}^{q_0} S_{BP}^{-1}(q)\,dq + \int_0^{\frac{B}{1-m}} j(q)\,S_{BP}^{-1}(q+q_0)\,dq\,, \tag{14}$$

with $q_0 = S_{BP}(c_0^-)$, $q_{BP} = J(m,B)$, $B = (1-m)(q_1 - q_0)$. Equation (14) includes the costs for the wholesale market and the BP markets. The first are determined by the costs of nBP suppliers in the interval $[0, D - q_{BP} - q_0]$ and the costs of BP suppliers in the interval $[0, q_0]$, while the costs for the BP markets are given by the costs of BP suppliers in the interval $[q_0, q_0 + \frac{B}{1-m}]$ weighted with the average active capacity of function $j(q)$.

3.5. Welfare Distribution

The producer surplus PS is given by

$$PS = D\, p_S + B^+\, p^+_{BP} + B^-\, p^-_{BP} - C\,.$$

PS includes the profits of the wholesale market $D\, p_S$ and of the BP payments $B^+\, p^+_{BP} + B^-\, p^-_{BP}$, while the total energy production costs C are subtracted. The BE costs do not effect PS because they are charged between the suppliers (see Section 3.2).

For the consumer surplus CS, we apply the concept of a consumers' reservation price (per energy unit) for a reliable power system p_{res} (e.g. Zolotarev, 2017), which leads to

$$CS = D\,(p_{res} - p_S) - B^+\, p^+_{BP} - B^-\, p^-_{BP}\,.$$

The consumer surplus CS incorporates the difference of the reservation price p_{res} and the wholesale market price p_S for the demand D. The BP costs $B^+\, p^+_{BP}$ and $B^-\, p^-_{BP}$ reduce CS because consumers bear the costs for keeping capacities available for BP.

Note that the variables p_S, B^+, p^+_{BP}, B^-, and p^-_{BP} have opposed effects on PS and CS: PS increases and CS decreases in each variable.

4. Market Equilibrium

The following propositions 1, 2, 3, and 4 are derived under the efficiency condition (A0), the micro-stability criteria (M1), (M2) and (M3), the conditions (S0), (S1), (S2) and (S3), symmetric BP markets, and a linear supply function

$$S(c) = \alpha\, c + \beta\,, \tag{15}$$

where $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$. Thus,

$$S_{BP}(c) = \delta(\alpha\, c + \beta)\,, \tag{16}$$

$$S_{nBP}(c) = (1-\delta)(\alpha\, c + \beta)\,. \tag{17}$$

4.1. Allocation, Costs and Prices

Proposition 1. *There exists an equilibrium of the wholesale market and the BP markets with the following prices:*

1. *Wholesale market price:* $p_S = \dfrac{D-\beta}{\alpha} \leq m\, c_1^+ + (1-m)\, c_0^+$
2. *BP price in the positive BP market:* $p_{BP}^+ = \dfrac{m}{1-m}\,(c_1^+ - p_S) = \dfrac{\frac{B}{2}\,\frac{m}{1-m}}{\delta\,\alpha}$
3. *BP price in the negative BP market:* $p_{BP}^- = 0$

See AppendixB for the proof . The wholesale market price p_S is determined by the inverse supply function at demand D. Condition $p_S \leq m\, c_1^+ + (1-m)\, c_0^+$ is necessary for stability since a higher p_S induces suppliers of positive BP to switch into the wholesale market. The power price p_{BP}^+ in the positive BP market exactly covers the wholesale market loss of the supplier with $c_1^+ > p_S$ caused by her costs of supplying the minimal load m and a rcf of zero. The power price p_{BP}^- in the negative BP market is zero (see also (M0)).

Proposition 2. *In the equilibrium of the wholesale market and both BP markets the following holds for c_0^-, c_1^+, c_0^-, and c_1^+:*

1. *The cost c_0^- of the supplier on the last rank in the negative BP market is given by*

$$c_0^- = \frac{D-\beta}{\alpha} - \frac{\frac{B}{2}\frac{1+m}{1-m}}{\delta\,\alpha} = p_S - \frac{\frac{B}{2}\frac{1+m}{1-m}}{\delta\,\alpha} \leq p_S.$$

2. *The costs c_1^- and c_0^+ of the suppliers on the first rank in both BP markets are given by*

$$c_1^- = c_0^+ = \frac{D-\beta}{\alpha} - \frac{\frac{B}{2}\frac{m}{1-m}}{\delta\,\alpha} = p_S - \frac{\frac{B}{2}\frac{m}{1-m}}{\delta\,\alpha} = p_S - p_{BP}^+ \leq p_S.$$

3. *The cost c_1^+ of the supplier on the last rank in the positive BP market is given by*

$$c_1^+ = \frac{D-\beta}{\alpha} + \frac{\frac{B}{2}}{\delta\,\alpha} = p_S + \frac{\frac{B}{2}}{\delta\,\alpha} \geq p_S.$$

See AppendixB for the proof. The cost c_0^- of the first BP supplier is determined by the difference between the wholesale market price p_S and the cost of the BP supply, which implies $c_0^- \leq p_S$. Costs c_1^- and c_0^+ are equal and are determined by the difference between p_S and the BP price in the positive BP market p_{BP}^+, which implies $c_1^- = c_0^+ \leq p_S$. The cost c_1^+ of the last BP supplier is determined by p_S and half of the costs of the entire BP supply, which implies $c_1^+ \geq p_S$. Hence, all suppliers of negative BP are inframarginal, i.e., their cost are below the p_S. This also applies to the suppliers of positive BP with low cost, while the suppliers of positive BP with higher cost are extramarginal, i.e., their cost are above p_S. According to Proposition 2 (2.), if the minimum load m converges to zero, the line between suppliers of negative BP and those of positive BP, given by c_1^-, converges to the line between inframarginal and extramarginal bidders, given by p_S. Thus, $c_1^- = c_0^+ = p_S$ for $m = 0$, i.e., all suppliers of negative BP are inframarginal and all suppliers of positive BP are extramarginal.

Proposition 3. *In the equilibrium of the wholesale market and both BP markets the following holds for the profits of the suppliers:*

1. *$\pi_S(c)$ and $\pi_{BP}(c)$ decrease in c.*
2. *$\pi_S(c) \geq 0$ for $c \in [\underline{c}, p_S]$ and $\pi_{BP}(c) \geq 0$ for $c \in [c_0^-, c_1^+]$.*
3. *$\pi_{BP}(c) \geq \pi_S(c)$, $\forall c \in [c_0^-, c_1^+]$.*

See AppendixB for the proof. Suppliers' profits decrease with their variable cost on all markets. The profits in all markets are (weakly) greater than zero, and a supplier's participation in the BP markets generates (weakly) higher profits than on the wholesale market.

Proposition 4. *The equilibrium of the wholesale market and both BP markets ensures overall market efficiency, i.e., it minimizes the total costs C of the power system.*

See AppendixB for the proof. Here is an intuitive explanation. An upward shift of the interval $[c_0^-, c_1^+]$ has three effects: more expensive power plants provide BP, the supply of BP suppliers on the wholesale market increases, which crowds out nBP suppliers on this market. In the case of symmetric BP markets and a linear supply function, the three cost effects cancel each other out in the equilibrium. The reasons are that, due to the symmetric BP markets, the costs of BP supply only depend on c_0^- and c_1^+ but not on $j(q)$, and, due to the linear supply function, the cost savings of negative BP (i.e., shutting down expensive power plants due to an overproduction of cheaper plants) equal the increasing costs for positive BP (i.e., activating more expensive plants due to an underproduction of cheaper plants).[11]

4.2. Effect of BP Prequalification

We now analyze the effects of less strict BP prequalification criteria, i.e., more flexibility for the suppliers to provide BP, on the equilibrium outcome, particularly on total costs (14). In our approach, suppliers' flexibility to offer BP is modelled by parameter δ, i.e., the share

[11]Note that in case of asymmetric BP markets or a non-linear supply function, efficiency cannot be guaranteed because the three cost effects do not necessarily cancel each other out in the equilibrium.

of prequalified BP suppliers. An increasing δ (i.e., more flexibility) has the following effects: By Proposition 1, p^+_{BP} decreases, while p_S and p^-_{BP} do not change. By Proposition 2, the interval $[c^-_0, c^+_1]$ becomes smaller because c^-_0 increases and c^+_1 decreases. The reduction of the interval length is caused by a higher density of BP suppliers. More precisely, the difference between $p_S = \frac{D-\beta}{\alpha}$ and c^-_0, between p_S and $c^-_1 = c^+_0$, and between c^+_1 and p_S decreases. What is the effect of a higher δ on the total costs (14)?

Proposition 5. *The total costs* (14) *decrease when δ increases.*

The proof is presented in AppendixB. With the focus on total costs, this results recommends to increase flexibility by lowering the BP prequalification criteria.

5. Comparison with German Market Data

In this section, we compare the German SBP market data with our theoretical results.

5.1. Parametrization of the Model

We calibrate our model to the German wholesale market and SBP markets in 2015.[12] Note that the results are presented in one hour with unit Euro/MWh, and that the current German positive and negative SBP markets are spilt into two time periods (regelleistung.net, 2017): a main period (peak) and a sub-period (offpeak).[13]

5.1.1. Demand

The total demand on the wholesale market was 525,000 GWh (BDEW and BMWi, 2016), which yields an average demand of $D = 59.93$ GW. The average demand on the positive and

[12]We investigate the market results of 2015 for two reasons: First, 2015 is the most recent year for which publicly available data for SBP was made available by the German TSOs. Second, since July 2016, the Austrian and German TSOs procure a common SBP merit-order, i.e., activation of SBP is linked within the two countries. Therefore, an analysis of the year 2016 had to include also the Austrian supply side.

[13]The main period includes Monday to Friday 8am to 8pm (60 hours per week), and the sub-period (offpeak) includes Monday to Friday 8am to 8pm as well as Saturday and Sunday (108 hours per week).

negative SBP market were $B^+ = 2.053$ GW and $B^- = 2.027$ GW and the average percentage demand were $\gamma^+ = 0.078$ and $\gamma^- = 0.060$ (BNetzA and BKartA, 2017). This yields average activated SBP capacities of $\tilde{B}^+ = 0.160$ GW and $\tilde{B}^- = 0.122$ GW.

5.1.2. Market Characteristics

The activated BE was 2,500 GWh in 2015. Hereof, 1,400 GWh were utilized for positive SBP and 1,100 GWh for negative SBP (BNetzA and BKartA, 2017). Thus, the demand for BE is asymmetric with 56% positive SBP and 44% negative SBP (44%), i.e., $a^+_{max} = 0.56$ and $a^-_{max} = 0.44$. We approximate these rcf by the exponential function

$$a(x(c)) = a_{max}\,\frac{e^{-tx(c)} - e^{-tx(c_1)}}{e^{-tx(c_0)} - e^{-tx(c_1)}} = a_{max}\,\frac{e^{-tx(c)} - e^{-t}}{1 - e^{-t}}\,. \tag{18}$$

This represents the normalized merit-order with $x(c) = \frac{c-c_0}{c_1-c_0} \in [0,1]$ for all $c \in [c_0, c_1]$, and gradient parameter t. Integrating $a(x(c))$ yields the percentage BE demand

$$\gamma = \int_0^1 a_{max}\,\frac{e^{-tx(c)} - e^{-t}}{1 - e^{-t}}\,dx = a_{max}\,\frac{1 - (t+1)\,e^{-t}}{t\,(1 - e^{-t})}\,.$$

Function (18) for the positive SBP market with $\gamma^+ = 0.078$, $a^+_{max} = 0.56$, $t^+ = 7.1$, and for the negative SBP market with $\gamma^- = 0.060$, $a^-_{max} = 0.44$, $t^- = 7.2$ are shown in Figure 3.

The (normalized) position in the merit-order $X(c)$ is given by

$$X(c) = \frac{\int_c^{c_1} x\,a(x(c))\,dx}{\int_c^{c_1} a(x(c))\,dx} \quad \forall c \in [c_0, c_1]\,,$$

which yields the (normalized) BE price $P(c)$

$$P(c) = (1 - \vartheta)\,X(c) + \vartheta \quad \forall c \in [c_0, c_1]\,.$$

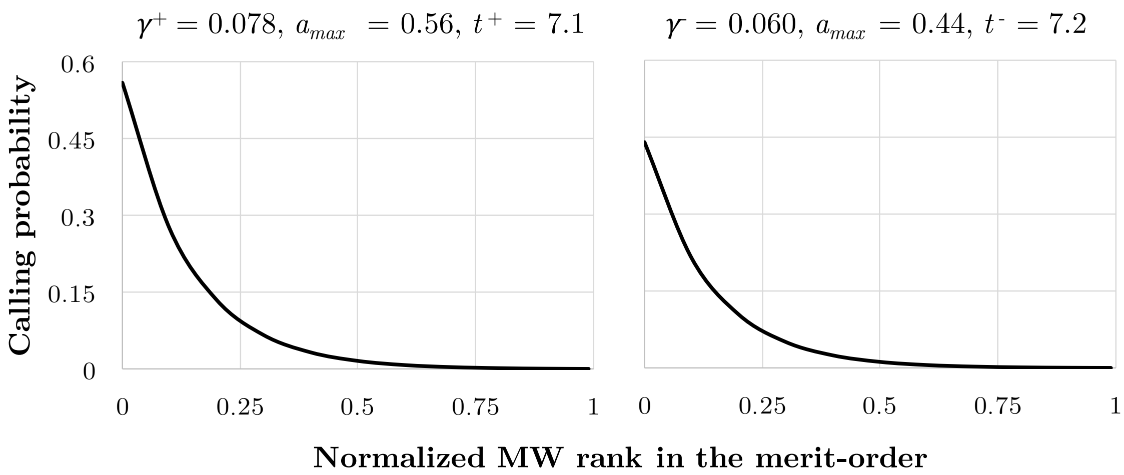

Figure 3: Approximation of the rcf in the German positive and negative SBP market.

The pricing rule in the German SBP market is PaB not UP. Since there is no publicly available information about the extent of the markups on the variable costs under PaB, we neglect markups, i.e., $\vartheta = 0$. Thus, our calculation represents a lower bound for the BE costs. This yields $X(c) = P(c) = 0.14$. Regarding must-run capacities, we assume $m = 0.5$ for all types of power plants (Steck and Mauch, 2008; Just and Weber, 2008; Hundt et al., 2009).

5.1.3. Supply Characteristics

For the wholesale market we refer to the Intraday Continuous Auction with an the average price $p_S = 33.31$ Euro/MWh in 2015 (Fraunhofer ISE, 2017).[14] We calibrate our model by this price p_S. We also consider renewable energy sources, which typically have variable cost close to zero and, thus, set the intercept of the supply function (Christoph et al., 2013; Nestle, 2014; Milojcic and Dyllong, 2016; Niedermeier et al., 2017). We use the official data on installed capacity provided by the German regulator (BNetzA, 2016). This yields $S(c) = 1.369\,c + 14.345$ in Figure 4, which includes the calibration point (33.31 Euro/MWh,

[14] The average price on the German Day Ahead Auction was 31.20 Euro/MWh, and 33.09 Euro/MWh on the Intraday Auction in 2015 (Fraunhofer ISE, 2017).

59,930 MW).[15] Since 2017, the German TSOs publish the prequalified capacities: 22.32 GW for positive SBP and 22.38 GW for negative SBP (regelleistung.net, 2017). We set $\delta = 0.22$.

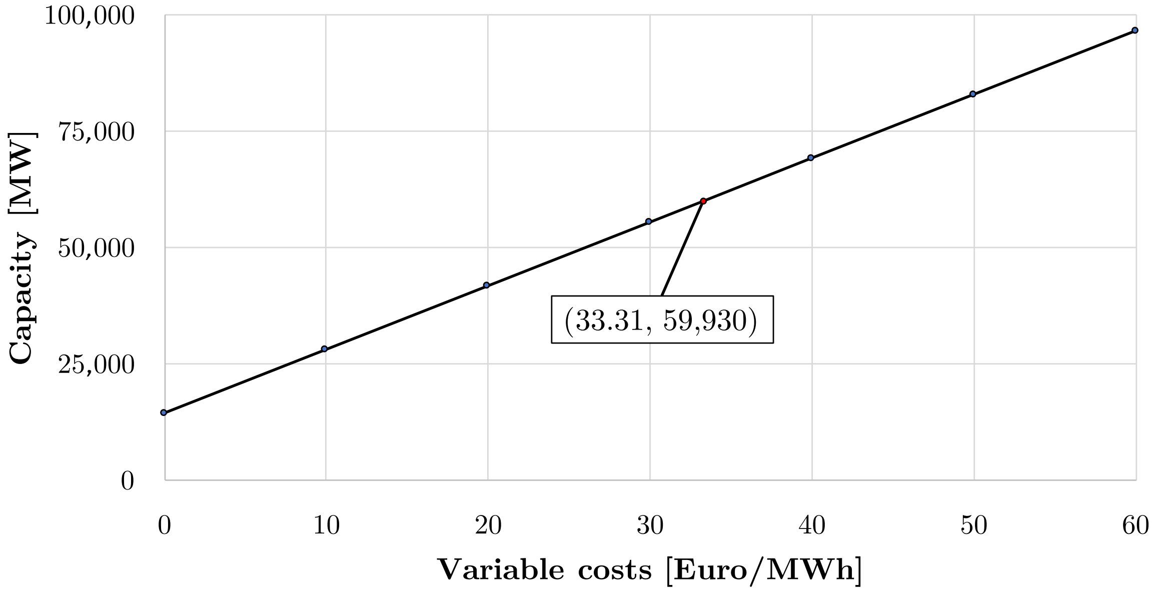

Figure 4: Linear approximation of the supply.

5.2. Results of the Integrated Model

The model results are presented in Tables 1 and 2.[16] The cost interval of the merit-order in the negative SBP market is [13.03, 26.49] Euro/MWh and [26.49, 40.13] Euro/MWh in the positive SBP market. All suppliers of negative SBP and about half of the suppliers of positive SBP have lower variable cost than the wholesale market price $p_S = 33.31$ Euro/MWh. The BP price in the positive SBP market is $p^+_{BP} = 6.82$ Euro/MWh and the BP price in the negative SBP market equals zero. The average BE price in the positive market is $p^+_{BE} = 28.40$ Euro/MWh and in the negative market $p^-_{BE} = -24.61$ Euro/MWh. The profits

[15]The linear supply function is an assumption in our model. It does not fully reflect the real-life market characteristics and should only be seen as an approximation of the actual supply function.

[16]The hourly cost of BE are 1,555 Euro. Thus, the BE cost for the suppliers are $dc = 0.026$ Euro/MWh.

at the interval boundaries are $\pi(c_0^-) = 20.28$, $\pi(c_1^- = c_0^+) = 6.82$, and $\pi(c_1^+) = 0.00$.[17]

Parameter	Model GER 2015
c_0^-	13.03
$c_1^- = c_0^+$	26.49
c_1^+	40.13
p_S	33.31
p_{BP}^+	6.82
p_{BP}^-	0.00
p_{BE}^+	28.40
p_{BE}^-	-24.61

Table 1: Merit-order position and prices in the markets with unit Euro/MWh.

Cost parameter	Model GER 2015
Total system	2,011,823
Wholesale market	1,996,268
SBP market	15,555
Positive SBP: BP	13,999
Positive SBP: BE	4,548
Negative SBP: BP	0
Negative SBP: BE	-2,992

Table 2: Hourly cost of the power system with unit Euro/h.

The total costs of the power system are 2,011,823 Euro/h. Hereof, over 99% of the costs are due to the wholesale market, and less than 1% due to the SBP market (positive and negative). The cost for positive SBP are 18,547 Euro/h and -2,992 Euro/h for negative SBP.[18]

Cost parameter	Real 2013	Real 2014	Real 2015	Model 2015
BP costs	**345**	**210**	**141**	**123**
Costs pos. BP	143	132	102	123
Costs neg. BP	202	78	39	0
BE costs	**58**	**50**	**64**	**14**
Costs pos. BE	95	65	72	40
Costs neg. BE	-37	-15	-8	-26
Total costs	**403**	**260**	**205**	**137**

Table 3: Empirical market results and asymmetric model results for the German SBP market of 2015 with unit Mio. Euro. Sources: regelleistung.net (2017); BNetzA and BKartA (2016, 2017).

5.3. Model Results and Empirical Market Results

For comparing our theoretic results with the actual data of 2013–2015, we convert the theoretic results into annual values and calibrate these according to the conditions in 2015 (see Table 3). Since the German TSOs do not provide detailed annual costs, we estimate the cost parameters based on the provided market data (regelleistung.net, 2017). That is, we estimate the average BP costs per week and convert them into annual values.[19]

The costs for BE depend on two parameters: the BE bids and the actual demand for BE.

[17] Results of the theoretical model for German SBP market of 2015 with $B^+ = B^- = 2.00$, $\gamma^+ = \gamma^- = 0.07$, and $a^+_{max} = a^-_{max} = 0.5$: $dc = 0.09$, $c^-_0 = 13.38$, $c^-_1 = c^+_0 = 26.67$, $c^+_1 = 39.95$, $p^+_{BP} = 6.64$, $p^-_{BP} = 0.00$, $p^+_{BE} = 28.53$, $p^-_{BE} = -24.81$, $\pi(c^-_0) = 19.93$, $\pi(c^-_1 = c^+_0) = 6.64$, and $\pi(c^+_1) = 0.00$

[18] Results of the theoretical model for German SBP market of 2015: hourly total system costs: 2,010,075, hourly wholesale market costs: 1,996,268, hourly SBP costs 13,807, hourly positive BP costs: 13,286, hourly negative BP costs: 0, hourly positive BE costs: 3,994, hourly negative BE costs: -3,473.

[19] For our estimations, we use the following data (BNetzA and BKartA, 2016, 2017):

		Positive SBP			Negative SBP		
		Avg. power bids [Euro/MW]		Avg. power	Avg. power bids [Euro/MW]		Avg. power
Year	Weeks	Main period	Sub period	demand [MW]	Main period	Sub period	demand [MW]
2013	52	552	740	2,122	726	1,142	2,081
2014	52	456	782	2,058	268	491	1,987
2015	53	350	585	2,053	102	267	2,027

We multiplied the average power bids in the main and sub period with the number of weeks and the average power demand for positive and negative SBP. Our estimates for the aggregated annual costs differ slightly from the official values: 353 Mio. Euro in 2013, 228 Mio. Euro in 2014, 155 Mio. Euro in 2015.

The BE bids are very volatile. However, if we assume for 2013 and 2014 the rcf of 2015 (see Figure 3), the average weighted BE bid of the activated plants is only slightly higher than the BE bid on the first position in the merit-order (less than 1 %). Thus, we approximate the BE costs by the average bid on the first position and convert these to annual values.[20]

The empirical results illustrate that almost all costs decreased. The total costs of 2013 (403 Mio. Euro) nearly halved in 2015 (205 Mio. Euro). The modelled result of 137 Mio. Euro indicates that there is still potential for further cost reductions. The total BP costs decreased from 345 Mio. Euro in 2013 to 141 Mio. Euro in 2015 although the BP demand was almost unchanged (BNetzA and BKartA, 2016, 2017). The model predicts 123 Mio. Euro. For the BP costs in the positive and negative SBP market, the differences between the actual and the modelled results are larger. While our model predicts costs of 123 Mio. Euro for positive BP in 2015, the actual costs were 102 Mio. Euro. The fact that the actual costs are lower than the predicted costs indicates that suppliers understate their opportunity costs in their power bids in favor of higher energy prices (Ocker et al., 2018b). That is, they submit low power bids to increase the award probability and, thus, to benefit from the high energy prices. The difference between the actual BE costs in the positive (negative) SBP market of 72 Mio. Euro (-8 Mio. Euro) and the prediction of 40 Mio. Euro (-26 Mio. Euro) gives support to this hypothesis. The BP costs in negative SBP market decreased more than 80% since 2013. In 2015 there is a difference of +39 Mio. Euro left between the actual and modelled results.

The development of the actual BE costs in the positive and negative SBP market differ.

[20]The average BE bid for positive SBP in the main period (sub-period) on the first position in the merit-order with unit Euro/MWh was: 67.79 (64.02) in 2013, 54.26 (56.57) in 2014, and 51.98 (50.50) in 2015. The average BE bid for negative SBP in the main period (sub-period) on the first position in the merit-order with unit Euro/MWh was: -22.20 (-12.51) in 2013, -16.20 (-5.30) in 2014, and -11.61 (-5.25) in 2015. We multiplied these numbers with the assigned hours per week for the main period (60h) and the sub-period (108h), the number of weeks: 52 weeks in 2013 and 2014, and 53 weeks in 2015 as well as with the average energy demand for positive (negative) SBP with unit MW: 166 (264) in 2013, 133 (184) in 2014, and 160 (122) in 2015.

While BE costs in the positive SBP market decreased from 95 to 72 Mio. Euro (with a small increase from 2014 to 2015), the BE costs in the negative SBP market increased from -37 to -8 Mio. Euro.[21] The large difference between the total BE cost level (50 to 64 Mio. Euro) and the prediction of 14 Mio. Euro is mainly due to the large difference of the BE costs in the positive SBP market (72 and 40 Mio. Euro), which we discussed before.[22]

5.4. Balancing Energy Prices

In this section we discuss empirical energy bids and the impact of the BEPP on BE prices.

5.4.1. Extreme Energy Bids

The empirical data reveal extreme bidding behavior on higher positions in the merit-orders, which is supported by recent studies (Heim and Götz, 2013; Ocker et al., 2018b; Ocker and Ehrhart, 2017; Ocker et al., 2018a). We illustrate the high energy bids for the merit-order positions 500, 1,000, and 1,500 of 2013, 2014, and 2015 in Table 4.

	2013			2014			2015		
	500	1,000	1,500	500	1,000	1,500	500	1,000	1,500
Main period (pos.)	103	164	242	94	164	349	84	159	447
Sub-period (pos.)	109	166	247	97	182	350	86	192	484
Main period (neg.)	-8	19	123	27	111	497	38	155	723
Sub-period (neg.)	7	49	203	31	119	435	51	220	841

Table 4: Average energy bids with unit Euro/MWh for merit-order positions 500, 1,000, and 1,500 in the German SBP markets (main period and sub-period) of 2013, 2014, 2015. Source: regelleistung.net (2017)

In the negative SBP market, bids lower than zero have disappeared entirely from position 500 onwards since 2014. Additionally, bids significantly increased over time on all positions.

[21] Note that the BE demand declines since 2013 (BNetzA and BKartA, 2016, 2017).

[22] The modelled costs of the symmetric German SBP market of 2015 with unit Mio. Euro are as follows: 116 for positive BP, 0 for negative BP, 35 for positive BE, -30 for negative BE, resulting in total costs of 121.

In the positive market, bids particularly increased on the last positions in the merit-order. Although the rcf substantially decreases on higher positions (see Figure 1), the empirical data illustrate the potential effect of increasing bids on costs.

5.4.2. Uniform Pricing and Length of the BEPP

The extremely high energy bids are accompanied by very low power bids (see Section 5.3). This reveals a major disadvantage of the applied scoring rule: since solely the power bid is relevant for winner determination, competition for energy bids is undermined, and facilitates the coordination on (extremely) high prices (Ocker et al., 2018b).

The European Commission (2017) implements UP in the future European SBP auction by arguing that it induces suppliers to report their true cost in their bids and, thus, leads to efficient outcomes. However, this reasoning is generally incorrect because the merit-order with ranked rcf induces the suppliers to underbid their true cost in the energy bids (Ocker et al., 2018a).[23] Only if the BEPP is short, suppliers are incentivized to truthfully report their cost in their energy bids.[24] Therefore, we advocate to set the BEPP to a short value.

However, we doubt that a switch to UP will fundamentally change suppliers' bidding behavior because this seems to be guided by other principles. There is empirical evidence that suppliers abused their market power and coordinated (i.e., implicitly colluded) on high price levels (Heim and Götz, 2013; Ocker and Ehrhart, 2017). Changing the pricing rule will not impede collusion as long as the design elements, which facilitate collusion, are not affected: the regular repetition of the auction with a limited and stable set of suppliers.[25]

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[23] Similar arguments hold for the position auctions of advertisements in search engines such as Google or Yahoo! (Varian, 2006; Edelman et al., 2007).

[24] If the BEPP is set to the smallest value, suppliers' payment for the energy bid is determined only by their energy bids, and therefore suppliers have an incentive to shade their bid as if PaB is applied (see Section 3.2).

[25] The European Commission (2017) also intends to introduce a voluntary energy bid market, i.e., suppliers that were not awarded within the regular SBP auction can submit an additional energy bid. This additional energy bid allows to be part of the merit-order without the power payment. This may increase the competition on the BE bids, and set an upper bound for energy bid payments (Ocker et al., 2018a).

If UP is applied, the length of the BEPP directly impacts the BE costs: the longer the BEPP, the higher is the position in the merit-order that determines the uniform price. Our model allows the comparison of different BEPP figures. The results of the model in Section 5.3 refer to a value of $\vartheta = 0$, i.e., PaB without markups. If we set $\vartheta = 1$ (the longest BEPP), we find that the annual SBP costs rise to 149 Mio. Euro, i.e., an increase of 9%.[26]

6. Conclusion

This paper examines the interaction of the electricity wholesale market and the BP markets. We develop an integrated market model under the condition that the BP suppliers run their plants at a minimal load and trade this energy on the wholesale market. In the overall market equilibrium, the suppliers are uniquely allocated to the different markets, e.g., all suppliers of negative BP are inframarginal (i.e., variable cost below the wholesale market price), while the suppliers of positive BP are either inframarginal or extramarginal. We also analyze the relationship between the prices on the different markets. These results may help to better understand the interplay between the markets. Moreover, we prove that the total costs of the power system decrease by lowering the BP prequalification criteria.

By comparing the empirical German market data with our theoretical results, we find that the BP costs decreased over the years and converged towards the theoretically predicted cost level. However, there are discrepancies, particularly with respect to the delivery costs of balancing energy: there are indications that suppliers systematically submit extremely high energy bids and very low power bids. This bidding pattern is an indicator for collusive behavior among the suppliers, which is facilitated by the fact that BP auctions are repeated on a regular basis and that the supply side is limited and (almost) invariant over time.

[26] Setting $\vartheta = 1$ yields $P(c) = 1$. The costs for BP reduce to 107 Mio. Euro, whereas the costs for BE increase to 42 Mio. Euro. The reason for this is the reduction of the BP price to 5.96 Euro/MWh, and an increase of the BE price in the positive (negative) market to 39.46 Euro/MWh (-12.36 Euro/MWh).

We are aware of the limitations of our theoretical analysis that may reduce its external validity. First, we assume a linear supply function, which does not reflect the real supply characteristics. Second, we model the same share of BP power plants across the entire supply. Third, the assumption of homogeneous must-run capacities across all types of power plants is a rough approximation. Finally, our integrated model could be applied to other European BP markets in order to evaluate auction outcomes more rigorously.

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References

50hertz, 2017. Regelenergie Downloadbereich. [accessed 31-October-2017].
URL http://www.50hertz.com/de/Maerkte/Regelenergie/Regelenergie-Downloadbereich

BDEW and BMWi, 2016. Nettostromverbrauch in Deutschland in den Jahren 1991 bis 2016 (in Terawattstunden). [accessed 31-October-2017].
URL https://de.statista.com/statistik/daten/studie/164149/umfrage/netto-stromverbrauch-in-deutschland-seit-1999/

BNetzA, 2016. Kraftwerksliste Bundesnetzagentur (bundesweit; alle Netz- und Umspannebenen) – Stand 10.05.2016. [accessed 20-May-2016].
URL https://www.bundesnetzagentur.de/DE/Sachgebiete/ElektrizitaetundGas/

Unternehmen_Institutionen/Versorgungssicherheit/Erzeugungskapazitaeten/Kraftwerksliste/kraftwerksliste-node.html

BNetzA and BKartA, 2016. Monitoringbericht 2015. [accessed 31-October-2017]. URL https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Allgemeines/Bundesnetzagentur/Publikationen/Berichte/2015/Monitoringbericht_2015_BA.pdf;jsessionid=1CFAAB5566000E812220C7431B6EFE8C?__blob=publicationFile&v=4

BNetzA and BKartA, 2017. Monitoringbericht 2016. [accessed 31-October-2017]. URL https://www.bundesnetzagentur.de/SharedDocs/Downloads/DE/Sachgebiete/Energie/Unternehmen_Institutionen/DatenaustauschUndMonitoring/Monitoring/Monitoringbericht2016.pdf?__blob=publicationFile&v=2

Borne, O., Korte, K., Perez, Y., Petit, M., Purkus, A., 2018. Barriers to entry in frequency-regulation services markets: Review of the status quo and options for improvements. Renewable and Sustainable Energy Reviews 81, 605–614.

Bushnell, J. B., Oren, S. S., 1995. Bidder cost revelation in electric power auctions. Journal of Regulatory Economics 6 (1), 5–26.

Chao, H.-P., Wilson, R., 2002. Multi-dimensional procurement auctions for power reserves: Robust incentive-compatible scoring and settlement rules. Journal of Regulatory Economics 22 (2), 161–183.

Christoph, K., Mayer, J., Thomsen, J., Hartmann, N., Senkpiel, C., Philipps, S., Nold, S., Lude, S., Schlegl, T., 2013. Stromentstehungskosten Erneuerbarer Energien. [accessed 02-November-2017], 1–48. URL https://www.ise.fraunhofer.de/content/dam/ise/de/documents/publications/studies/DE2013_ISE_Studie_Stromgestehungskosten_Erneuerbare_Energien.pdf

Edelman, B., Ostrovsky, M., Schwarz, M., 2007. Internet advertising and the generalized second-price auction: selling billions of dollars worth of keywords. The American Economic Review 97(1), 242–253.

Engelbrecht-Wiggans, R., 1988. Revenue equivalence in multi-object auctions. Economics Letters 26 (1), 15–19.

entso-e, 2016. Survey on ancillary services procurement and electricity balancing market design. [accessed 22-August-2017].
URL https://www.entsoe.eu/publications/market-reports/ancillary-services-survey/Pages/default.aspx

European Commission, 2017. Electricity balancing guideline. Commission Regulation (EU) 2017/2195 of 23 November 2017 establishing a guideline on electricity balancing [accessed 13-December-2017].
URL http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32017R2195

Fraunhofer ISE, 2017. Annual electricity spot market prices in Germany. [accessed 03-November-2017].
URL https://www.energy-charts.de/price_avg.htm?year=2015&price=nominal&period=annual

Heim, S., Götz, G., 2013. Do pay-as-bid auctions favor collusion? Evidence from Germany's market for reserve power. ZEW - Centre for European Economic Research Discussion Paper No. 35.

Hirth, L., Ziegenhagen, I., 2015. Balancing power and variable renewables: Three links. Renewable and Sustainable Energy Reviews 50, 1035–1051.

Hundt, M., Barth, R., Sun, N., Wissel, S., Voss, A., 2009. Verträglichkeit von erneuerbaren

Energien und Kernenergie im Erzeugungsportfolio – Technische und ökonomische Aspekte. [accessed 31-October-2017].
URL http://www.ier.uni-stuttgart.de/publikationen/pb_pdf/Hundt_EEKE_Langfassung.pdf

Just, S., Weber, C., 2008. Pricing of reserves: Valuing system reserve capacity against spot prices in electricity markets. Energy Economics 30, 3198–3221.

Kahn, A. E., Cramton, P. C., Porter, R. H., Tabors, R. D., 2001. Uniform pricing or pay-as-bid pricing: a dilemma for California and beyond. The Electricity Journal 14 (6), 70–79.

Kamat, R., Oren, S. S., 2002. Rational buyer meets rational seller: Reserves market equilibria under alternative auction designs. Journal of Regulatory Economics 21 (3), 247–288.

Kaut, A., Obermüller, F., Weiser, F., 2017. Tender frequency and market concentration in balancing power markets. Institute of Energy Economics at the University of Cologne (EWI) Working Paper 17/04, 242–259.

Krishna, V., 2002. Auction theory. Academic Press.

KU Leuven Energy Institute, 2015. The current electricity market design in Europe. [accessed 14-November-2017].
URL https://www.google.at/url?sa=t&rct=j&q=&esrc=s&source=web&cd=4&ved=0ahUKEwifh_3r3L3XAhVHoaQKHYlFAeAQFghEMAM&url=https%3A%2F%2Fset.kuleuven.be%2Fei%2Fimages%2FEI_factsheet8_eng.pdf%2Fat_download%2Ffile&usg=AOvVaw3Ju_fQUbcOG6ZKEdJzTU2c

Milojcic, G., Dyllong, Y., 2016. Vergleich der Flexibilität und der CO2-Emissionen von Kohlen- und Gaskraftwerken. energiewirtschaftliche tagesfragen 07/2016, 29–31.

Müller, G., Rammerstorfer, M., 2008. A theoretical analysis of procurement auctions for tertiary control in Germany. Energy Policy 36, 2620–2627.

Müsgens, F., Ockenfels, A., Peek, M., 2014. Economics and design of balancing power markets in Germany. International Journal of Electrical Power & Energy Systems 55, 392–401.

Nestle, U., 2014. Stromentstehungskosten verschiedener Erzeugungstechnologien. [accessed 02-November-2017], 1–16.
URL http://www.forschungsradar.de/fileadmin/content/bilder/Vergleichsgrafiken/Stromgestehungskosten_okt2014/AEE_Dossier_Studienvergleich_Stromgestehungskosten_sep14.pdf

Niedermeier, R., Bruckner, T., Pels Leusden, C., 2017. Die Vermessung der Auction-Curves - ein Ansatz zur Analyse der Stunden-Spotpreise der EPEX. Proceedings of the tenth IEWT Conference 2017 07/2016, 1–24.

Ocker, F., 2017. Design and performance of European balancing power auctions. Proceedings of the 14th International Conference on the European Energy Market, 1–6.

Ocker, F., Braun, S., Will, C., 2016. Design of European balancing power markets. Proceedings of the 13th International Conference on the European Energy Market, 1–6.

Ocker, F., Ehrhart, K.-M., 2017. The "German paradox" in the balancing power markets. Renewable and Sustainable Energy Reviews 67, 892–898.

Ocker, F., Ehrhart, K.-M., Belica, M., 2018a. Harmonization of the European Balancing Power Auction: A Game-theoretical and Empirical Investigation. Energy Economics 73C, 194–211.

Ocker, F., Ehrhart, K.-M., Ott, M., 2018b. Bidding strategies in the Austrian and German secondary balancing power market. Wiley Interdisciplinary Reviews - Energy and Environment 7(6), e303, 1–16.

Ortner, A., 2017. Electricity balancing markets in Europe – fundamental modeling approaches and case studies assuming perfect competition. Dissertation TU Vienna.

Rammerstorfer, M., Wagner, C., 2009. Reforming minute reserve policy in Germany: A step towards efficient markets? Energy Policy 37, 3513–3519.

regelleistung.net, 2017. Internetplattform zur Vergabe von Regelleistung. [accessed 12-January-2017].
URL `https://www.regelleistung.net`

Steck, M., Mauch, W., 2008. Technische Anforderungen an neue Kraftwerke im Umfeld dezentraler Stromerzeugung. 10. Symposium Energy Innovation, 1–13.

Ströbele, W., Pfaffenberger, W., Heuterkes, M., 2013. Energiewirtschaft: Einführung in Theorie und Politik. Oldenburg (Munich).

Swider, D. J., Weber, J., 2007. Bidding under price uncertainty in multi-unit pay-as-bid procurement auctions for power systems reserve. European Journal of Operational Research 181, 1297–1308.

Varian, H. R., 2006. Position auctions. International Journal of Industrial Organization 25, 1163–1178.

Wen, F., David, A., 2002. Coordination of bidding strategies in day-ahead energy and spinning reserve markets. International Journal of Electrical Power & Energy Systems 24 (4), 251–261.

Zolotarev, P., 2017. Social welfare of balancing markets. Proceedings of the 14th International Conference on the European Energy Market, 1–6.

Zweifel, P., Praktiknjo, A., Erdmann, G., 2017. Energy Economics. Springer (Berlin).

AppendixA. Notation

α	gradient of function S
β	intercept of function S
$\gamma^{+/-}$	fraction of provided positive (negative) BP
δ	share of BP suppliers
ϑ	length of the BEPP
π	supplier's profit function
$a^{+/-}$	relative calling frequency (abbrev.: rcf) in the positive (negative) BP market
$a^{+/-}_{max}$	highest rcf in the positive (negative) market
$a^{+/-}_{min}$	lowest rcf in the positive (negative) market
B	capacity demand on the positive and negative BP market
$B^{+/-}$	capacity demand on the positive (negative) BP market
$\tilde{B}^{+/-}$	expected positive (negative) capacity for balancing excess supply and demand
C	total cost function of the power system
$[\underline{c}, \overline{c}]$	interval of the variable energy production costs
$c^{+/-}_0$	lowest variable cost of all suppliers on the positive (negative) BP market
$c^{+/-}_1$	highest variable cost of all suppliers on the positive (negative) BP market
$c + dc$	imputed variable cost
CS	consumer surplus
D	average demand on the wholesale market
j	function of BP bidders' average active capacities
J	BP bidders' average active capacities (integral of j)

m	minimal load capacity
P	normalized BE price
p_S	wholesale market price
$p_{BP}^{+/-}$	BP price in the positive (negative) market
$p_{BE}^{+/-}$	BE price in the positive (negative) market
p_{res}	reservation price for a reliable power system
PS	producer surplus
$R^{+/-}$	set of positive (negative) BP merit-order ranks
$r^{+/-}$	rank function in the positive (negative) BP merit-order
X	normalized position in the merit-order
S	supply function
S_{BP}	supply function of BP suppliers
S_{nBP}	supply function of nBP suppliers
z	function of relative frequency of the differences between demand and supply

AppendixB. Proofs

Proof of Proposition 1. The existence of the equilibrium and 1., 2., and 3. follow by solving the equation system given by (M0), (M1), (M2), (S0), (S1), (S2), (S3) and $S(c) = \alpha\, c + \beta$. Alternatively, 3. follows directly from (M1).

Proof of Proposition 2. 1., 2., and 3. follow directly by solving the equation system given by (M0), (M1), (M2), (S0), (S1), (S2), (S3), symmetric BP markets and $S(c) = \alpha\, c + \beta$. The restriction $p_S \leq m\, c_1^+ + (1-m)\, c_0^+$ is implied by $\pi_S(c) - \pi_{BP}(c) \leq 0$, which yields

$$p_S - c \leq \frac{m}{1-m}(c_1^+ - p_S) + a(r(c))\,\vartheta\,(c_1^+ - c)\,.$$

Now consider $\vartheta = 0$, and the condition $c \geq c_0^+$. This yields

$$p_S \leq m\, c_1^+ + (1-m)\, c_0^+ \,,$$

which is a sufficient condition for the claim.

Proof of Proposition 3.

1. The derivative of (7) is

$$\frac{\partial\, \pi_{BP}^+(c)}{\partial\, c} = -m + (1-m)\,(a'(r(c))\, r'(c)\, \vartheta\, (c_1^+ - c) - a(r(c))\, \vartheta) < 0 \,,$$

   and is strictly decreasing in c, with $a'(r(c)) < 0$, $r'(c) > 0$. Differentiation (8) yields

$$\frac{\partial\, \pi_{BP}^-(c)}{\partial\, c} = -1 + (1-m)[a'(r(c))\, r'(c)\, \vartheta\, (c - c_0^-) + a(r(c))\, \vartheta] < 0 \,,$$

   and is strictly decreasing in c, with $a'(r(c)) < 0$, $r'(c) > 0$.

2. Equation (6) directly implies that $\pi_S \geq 0$ for all $c \in [\underline{c}, p_S]$. In the positive market, reformulating (7) with $p_{BP}^+ = \frac{m}{1-m}(c_1^+ - p_S)$ immediately yields the result. For a stable market equilibrium, (M2) demands that $\pi_{BP}^-(c_1^-) = \pi_{BP}^+(c_0^+)$, with $\pi_{BP}^+(c_0^+) \geq 0$. Since the profits of all BP suppliers decrease in c, all profits in the negative BP market must be (weakly) greater than zero.

3. Consider the suppliers with variable cost $c \in [c_0^-, c_1^-]$. The proposition demands $\pi_S(c) - \pi_{BP}(c) \leq 0$, which is immediately implied by straightforward computation. For the suppliers with variable cost $c \in [c_0^+, c_1^+]$, see the proof of Proposition 2.

Proof of Proposition 4. The derivative of (14) with respect to q_0 is

$$\begin{aligned}\frac{\partial C}{\partial q_0} &= -S_{nBP}^{-1}\left(D - \frac{1+m}{1-m}\,\frac{B}{2} - q_0\right) + S_{BP}^{-1}(q_0) + \int\limits_0^{\frac{B}{1-m}} j(q)\, S_{BP}^{\prime -1}(q_0+q)\, dq \\ &= -\frac{D - \frac{B}{2}\,\frac{1+m}{1-m} - q_0 - \beta\,(1-\delta)}{\alpha\,(1-\delta)} + \frac{q_0 - \delta\,\beta}{\delta\,\alpha} + \frac{B\,(1+m)}{2\,\alpha\,\delta\,(1-m)} \overset{!}{=} 0\,,\end{aligned}$$

which yields

$$q_0^* = D\,\delta - \frac{B\,(1+m)}{2\,(1-m)}\,.$$

Reformulating shows that c_0^{*-} equals the equilibrium c_0^- in Proposition 2

$$c_0^{*-} = \frac{D-\beta}{\alpha} - \frac{\frac{B}{2}\,\frac{1+m}{1-m}}{\delta\,\alpha}\,.$$

Proof of Proposition 5. By Proposition 1, $p_s = \frac{D-\beta}{\alpha}$ in the equilibrium leads to

$$D - q_{BP} - q_0 = (1-\delta)D\,. \tag{B.1}$$

By (13) and (14) ,

$$q_{BP} = J(m,B) = \frac{B(1+m)}{2(1-m)}\,. \tag{B.2}$$

Thus, by (B.1) and (B.2),

$$q_0 = \delta D - q_{BP} = \delta D - \frac{B(1+m)}{2(1-m)}\,, \tag{B.3}$$

$$\frac{\partial q_0}{\partial \delta} = D\,. \tag{B.4}$$

With (16), (17), and (B.1) the total costs (14) can be written as

$$C = C_{nBP} + C_{BP} + C_J \,,$$

$$C_{nBP} = \int_{(1-\delta)\beta}^{(1-\delta)D} \frac{q}{(1-\delta)\alpha} - \frac{\beta}{\alpha} dq \,, \tag{B.5}$$

$$C_{BP} = \int_{\delta\beta}^{q_0} \frac{q}{\delta\alpha} - \frac{\beta}{\alpha} dq \,, \tag{B.6}$$

$$C_J = \int_0^{\frac{B}{1-m}} j(q) \left(\frac{q+q_0}{\delta\alpha} - \frac{\beta}{\alpha} \right) dq \,. \tag{B.7}$$

We have to prove

$$\frac{\partial C}{\partial \delta} = \frac{\partial C_{BP}}{\partial \delta} + \frac{\partial C_{nBp}}{\partial \delta} + \frac{\partial C_J}{\partial \delta} < 0 \,. \tag{B.8}$$

With (2) and (B.2) – (B.7),

$$\begin{aligned}
\frac{\partial C_{nBP}}{\partial \delta} &= -\frac{(D-\beta)^2}{2\alpha} \,, \\
\frac{\partial C_{BP}}{\partial \delta} &= \frac{(D-\beta)^2}{2\alpha} - \frac{q_{BP}^2}{2\delta^2\alpha} \,, \\
\frac{\partial C_J}{\partial \delta} &= \int_0^{\frac{B}{1-m}} j(q) \left(\frac{q_{BP}-q}{\delta^2\alpha} \right) dq = \frac{q_{BP}^2}{\delta^2\alpha} - \frac{1}{\delta^2\alpha} \int_0^{\frac{B}{1-m}} j(q) q \, dq \,,
\end{aligned}$$

which in (B.8) together with (B.2) yield

$$\frac{\partial C}{\partial \delta} = \frac{1}{\delta^2\alpha} \left(\frac{q_{BP}^2}{2} - \int_0^{\frac{B}{1-m}} j(q) q \, dq \right) = \frac{1}{\delta^2\alpha} \left(\frac{B^2(1+m)^2}{8(1-m)^2} - \int_0^{\frac{B}{1-m}} j(q) q \, dq \right) < 0 \,. \tag{B.9}$$

To prove (B.9) we consider the j-function

$$j_0(q) = \begin{cases} 1 & : q \in [0, \frac{B}{2(1-m)}) \\ m & : q \in [\frac{B}{2(1-m)}, \frac{B}{1-m}] \,, \end{cases} \tag{B.10}$$

which fulfils the requirements of a j-function (9) and $J(m, B) = \frac{B(1+m)}{2(1-m)}$. Since (B.10) assigns the highest possible j-value (i.e., 1) to the low q-values in $[0, \frac{B}{2(1-m)})$ and the lowest possible j-value (i.e., m) to the high q-values in $[\frac{B}{2(1-m)}, \frac{B}{1-m}]$, (B.10) is the j-function with lowest integral $\int_0^{\frac{B}{1-m}} j(q) q \, dq$ in the class of j-functions (9) with $J(m, B) = \frac{B(1+m)}{2(1-m)}$. That is, for all j-functions $j(q)$ in this class:

$$\int_0^{\frac{B}{1-m}} j(q) q \, dq \geq \int_0^{\frac{B}{1-m}} j_0(q) q \, dq = \frac{B^2(1+3m)}{8(1-m)^2} \,. \tag{B.11}$$

Since

$$\frac{B^2(1+3m)}{8(1-m)^2} > \frac{B^2(1+m)^2}{8(1-m)^2} \;\Leftrightarrow\; 1 > m > 0 \,, \tag{B.12}$$

Condition B.9 is fulfilled and, thus, also Condition (B.8), which completes the proof.